A Theoretical Model for Measuring Stress Induced by a Vibrating load at Ice/Material Interface

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***Abstract*--The aim of this paper is to present theoretical investigations introducing a new technique for the purpose of measuring atmospheric-ice adhesion. This research is focusing on the development of a macroscopic and direct technique for measuring ice adhesion using embedded piezoelectric sensors. Other mechanical methods for measuring ice adhesion as well as factors influencing such measurements are briefly reviewed. The advantages of the present method over these are discussed. The analytical solution of a non-symmetric bimorph constituted by a layer of atmospheric-ice deposited on an elastic substrate, driven into harmonic vibration is obtained. Analytical and theoretical equations of shear and bending stresses due to a vibrating load in a cantilever bimorph are derived. The piezoelectric charge coefficient is used to predict the charge density induced on the piezoelectric film. The discrepancies in Young moduli and the geometric dimensions of the two layers are taken into account in the modeling.**

I. NOMENCLATURE

Atmospheric-ice, ice adhesion strength, cohesive and adhesive failure, PVDF films

II. INTRODUCTION

THE unprecedented January 1998 icing storm that hit wide areas of Quebec, Ontario and the Maritimes clearly illustrates the severity of the problems possibly resulting from atmospheric ice accretion on overhead power lines, such as structural damage and electrical outages, with related consequences for the affected utilities and populations [1]. The removal of ice deposits using anti-icing or ice phobic materials has not yet been achieved. The development of such techniques requires a greater knowledge of the physical-mechanical phenomena at the ice/material interface. Moreover, the determination of ice adhesion strength is very important for selecting more efficient and economical de-icing techniques and for identifying materials with low surface energy. It is equally important for determining the mechanical energy necessary, at the ice/material interface, for ice removal. In fact, few studies have focused on the basic mechanism underneath ice adhesion because of its high sensitivity to test conditions, such as ice type and structure, temperature and test techniques. To the best of our knowledge, ice adhesion measurement has been attempted in a variety of ways, but the results are scattered and difficult to compare [2]. Consequently, this study is necessary because of the importance of developing effective ice accumulation prevention methods and undertaking more research on ice adhesion.

Because of the brittle structure of ice, the known methods in the general field of adhesion measurement are difficult to apply. However, with the development of smart/active materials like PVDF films as sensors, which allows the reversible transformation of mechanical energy into electric power, studying the mechanical behavior of materials at the interfaces is easier.

The aim of this study is to obtain theoretical equations of stress induced by a vibrating load at the ice/material interface. Using embedded piezoelectric sensors enables us to develop a macroscopic and direct measurement technique for determining mechanical stresses at the atmospheric-ice/substrate interface.

In what follows, the next section will be devoted to a brief review of different mechanical and macroscopic methods for measuring ice adhesion, and of factors influencing ice adhesion strength. In section IV, there will be a description of the setup geometry, modeling of structure and its advantages in comparison with the existing methods. In section V, the analytical modeling and the theoretical equations of shear and bending stresses for this configuration will be presented. In the final section, the advantages of using piezoelectric films (PVDF) as stress sensors will be listed and related sensor equations for predicting the charge density induced on the piezoelectric films will be explained.

III. TECHNICAL REVIEW

Few mechanical methods make it possible to measure the mechanical adhesion force of ice on different materials [2]. The reason is that, in the range of ordinary forces, ice is a brittle material and its cohesion force is weaker than its adhesion force, thus making cohesive failure is more likely. A simple tensile test with stress applied normally to the interface frequently results in a fracture within the ice, known as 'cohesive break'. On the other hand, an 'adhesive break', occurring on the interface itself, is often observed when a shear stress is applied in direction of the plane of the interface. This geometry is therefore more commonly used in the study of ice adhesion [3]. In a previous study, a complete review of ice adhesion measurement methods has been carried out including both macroscopic and microscopic/nanoscopic scale tests [2]. They are also compared as to their performances and limitations. However, a brief review of the existing mechanical methods seems necessary for a general overview.

Adhesion strength is energy per unit surface required to separate ice from its substrate in adhesive failure. The basic of an adhesion test involve a load applied to adhesive/substrate system until failure occurs. Shear mode tests [4], such as the "Lap Shear Test" or, "Torsion Shear Test" are more common. In the Torsion Shear Test the adhesive layer is formed between a fixed plate and a flat disk, which is rotated, with measurement of force for failure. In "Cylinder Torsion Shear Test" the ice freezes between a hollow cylinder and central core, one of which is rotated and the torque measured. In "Axial Cylinder Shear Test", for which the ice is deposited between a hollow cylinder and central core, the applied force is axial. Fig. 1, on left-hand side exhibits the "Cone Test" where cone angle $0^0 < \phi < 90^0$. H1 is application of axial force which results in combination of tensile and shear stress. H2 is application of torsion force which results in shear stress within the ice. When $\phi = 90^0$ it will result as the Lap Shear Test While $\phi = 0^0$ it can give two different results. By applying torsion force, H_2, it results as the same as the Cylinder Torsion Test, and with axial force, H_1, it results as the same as the Axial Cylinder Shear Test [4].There are other techniques like "Tensile Tests" and "Peel Tests" where the ice is frozen onto a rigid base and an overlapping free end is pulled away at right angles, measuring the force. In "Impact Test" a load is applied suddenly on the surface of deposited ice till ice detachment. Fig. 1, on right-hand side shows the "Blister Test" in which a pressure is applied at the center of the adhering interface, the thinner and more flexible member of which is thereby debond in blister-fashion with measuring the adhesion force. Another method uses a beam on which ice is deposited [5]. An instantaneous pressure is applied to the beam by a mechanical impulse which leads to ice separation. Ice thickness is chosen in order to induce the maximum value of shear stress at the interface which is indirectly deduced from measuring the applied force and beam displacement. Since in this method the thickness of the ice coating is selected to the value for which the neutral axis locate at vicinity of the ice/aluminum interface that will be a limiting method. The high sensitivity to the thickness of ice and insensitivity to measure weak adhesion forces, as in the case of ice-phobic materials, can be lead into some inaccuracies.

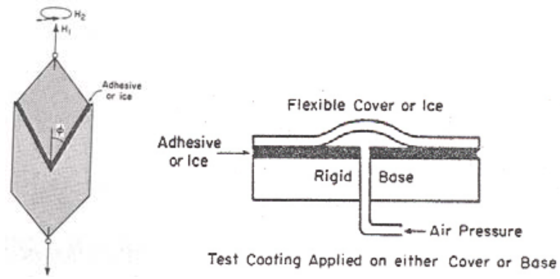

Fig. 1. The Cone Test where cone angle $0^0 < \phi < 90^0$ and The Blister Test, two techniques for measuring ice adhesion

It is found [4] that in some of the enumerated cases increasing rate of applied load is instantaneous and/or the load is applied directly on the ice which has a brittle structure. It is to be noted that different results may be anticipated using different methods. For example, tensile experiments showed that the adhesive strength under tension is at least 15 times larger than that from shear experiments [4]. It has been found that adhesive strength depends on the following parameters: rate of loading; substrate characteristics (Young modulus, cohesive strength of substrate, surface finish, and surface energy); temperature; type of Ice; and how the load is applied. Another factor to consider is ice expansion during the solidification process, which could also affect some of the test schemes. In order to obtain a higher degree of reproducibility the solidification processe require a long time, e.g., about eight hours to freeze the ice sample completely and about forty hours to be relaxed the internal stresses before testing. Indeed, it has been shown that paying attention to sample growth and test conditions leads to reproducible results [6].

IV. BIMORPH CHARACTERIZATION AND MODELING

Fig.2 shows the configuration used for modeling. The aluminum beam which is used as substrate, has an edge clamped and the other free, and is characterized by the following dimensions: h_s thick (z coordinate), b wide (y coordinate) and L long (x coordinate). The ice layer is deposited on this alumina substrate with the same length and width. A vibrating load is applied at the free end as shown in Fig.2. The coordinate x axis passes through the ice/material interface. The Piezoelectric films can be placed on the aluminum surface before ice deposition. As stress distribution varies along the beam and is a function of x, then the piezoelectric films length is set relatively small in order to sense homogenous stress while their width is the same as the beam. Assuming that the piezoelectric film thickness is very thin compared to the substrate and ice thickness. Hence, the mechanical influence of the piezoelectric film has not taken into account. One of the main advantages of this configuration is that the thickness of the ice or substrate is not confined by the position of the neutral axis, as in the case of the presented method in [5]. In this configuration, adhesive failure is more likely, because of the creation of the shear stress at interface. The rate of applied load is incremental and the load is applied on the substrate surface not on the ice directly which has a brittle structure.

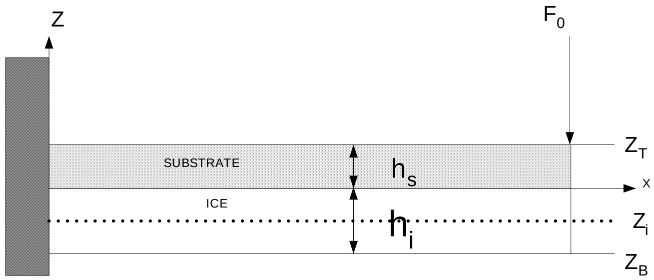

Fig. 2 Geometry and coordinates of the model configuration

A harmonic vibration is subjected to this structure that results deflection $w(x,y,z,t)$. For this modeling, two important assumptions are made. First the beam thickness is supposed to be very small compared to its length, so that strain and stress along the z axis are equal to zero. Second, the structure width is assumed small compared to its length, hence strain and stress along the y-direction is also equal to zero. So the bending displacement or deflection $w(x,y,z,t)$ reduces to $w(x,t)$. This displacement occurs along z-direction and is a function of the x coordinate. Assuming that the bimorph undergoes small deflections in the linearly elastic region, and has a uniform cross-section then the differential equation of motion for the bending displacement of the beam is known. Rotary inertia effects have not been considered. The bending displacement is obtained of the moment equilibrium in the structure. According to the above assumptions, the bending displacement along the z-axis can be written as

$$w(x,t) = W(x)\cos\omega t \quad (1)$$

Or the real part of

$$w(x,t) = W(x)\exp j\omega t \quad (2)$$

This beam transmits a shear force and a bending moment. Note that the x axis must be replaced from the plane of interface and located along the undeformed neutral axis of the beam. Therefore the first step is determining the neutral plane position.

V. ANALYTICAL MODELING AND THEORETICAL EQUATIONS OF SHEAR STRESS AND BENDING STRESS

A. Determination of Neutral Plane Position

When a beam is bent one side narrows and the other side lengthens. Between the two, there is a neutral plane for which the bending stress is equal to zero and shear stress has its maximum value. The coordinate origin, the x axis, must be located at the neutral plane position in order to have the right value of z for calculating both bending and shear stress. The neutral plane position can be calculated using the moment equilibrium of the structure. It is carried out by calculating the stress in a cross section of the bimorph. The neutral plane is located at an unknown distance z_i from the interface as it is showed in Fig.2. Fig.3 shows the bending stress repartition in a longitudinal element of the beam having a length of dL.

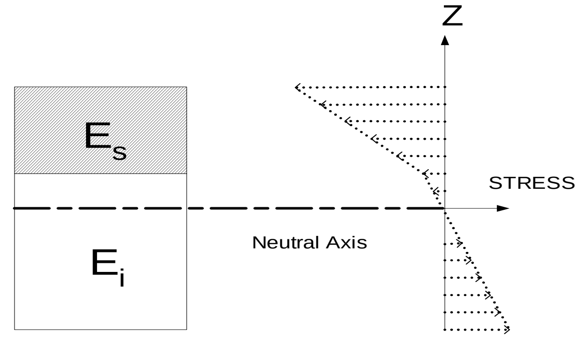

Fig. 3 Bending stress repartition in an element

In each material, the stress variation versus the z coordinate is linear and the element equilibrium requires the sum of normal forces to go to zero [7],

$$F_x = \int \sigma_x dA = 0 \quad (3)$$

where σ_x is the bending stress. Considering a section made of two different materials with Young's modulus Y_i and Y_s shown in Fig.3,

$$\int_{-z_B}^{z_i} \sigma_{ix} dA + \int_{z_i}^{z_T} \sigma_{sx} dA = 0 \quad (4)$$

Since $\sigma_x = -\frac{Mz}{I}$, by consideration $|z|_{\max} = c$ we have, $\sigma_x = \sigma_{\max}\frac{z}{c}$, substituting in (4) gives,

$$E_i \int_{-z_B}^{z_i} (\frac{z}{c} S_{\max}) dA + E_s \int_{z_i}^{z_T} (\frac{z}{c} S_{\max}) dA = 0 \quad (5)$$

where $S_{\max}$ is the maximum strain at surfaces as shown in Fig. 4, and E_s, E_i are the Young's modulus of the substrate and ice. After simplification,

$$z_B^2 = (1 - E_s / E_i) z_i^2 + (E_s / E_i) z_T^2 \quad (6)$$

Assuming the neutral axis is located within the ice; the ice and substrate thicknesses are known, then substitution of $z_B = h_i - z_i$ and $z_T = h_s + z_i$ in (6) gives,

$$z_i = \frac{h_i^2 - (E_s / E_i) h_s^2}{2h_i + 2(E_s / E_i) h_s} \quad (7)$$

where h_i is ice thickness and h_s is substrate (AL) thickness. If the obtained value for z_i is negative, it means that the NA is located in substrate. As it was expected in the case of having one material the NA is located at the mid plane. The coordinate origin, x axis will now be located at the coordinate z_i from the ice/material interface.

B. Determination of Deflection (Bending Displacement)

Assuming that along the beam, the elastic moduli, inertia, and cross section area are constant. A vibrating force is applied at the free end of bimorph on the free surface; with constant amplitude have a form of $f(L,t) = F_0 \exp(j\omega t)$. The differential equation of motion for the bending displacement of the beam (Euler-Bernoulli Beam theory) is given by [8]-[9]-[10]-[11]:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}+\rho A\frac{\partial^2 w(x,t)}{\partial t^2}=f(L,t) \qquad (8)$$

where EI is the global flexural rigidity, which is given by (22) and ρ is the total density $\rho=\rho_i+\rho_s$ where ρ_i and ρ_s are ice and substrate densities, respectively, and A is the cross section area, $A=b*(h_i+h_s)$. A separation of variables is assumed of the form $w(x,t)=W(x)T(t)$ [11]. The solution of temporal equation gives ,

$$T(t)=T_1\cos(\omega t)+T_2\sin(\omega t) \qquad (9)$$

The coefficients T_1 and T_2 can be calculated only if two initial conditions (initial deflection and velocity) are specified. The spatial equation comes [10],

$$EI\frac{d^4W(x)}{dx^4}-\rho A\omega^2 W(x)=0 \qquad (10)$$

Or

$$\frac{d^4W}{dx^4}-\beta^4 W=0 \qquad (11)$$

where β is the flexural wave number given by

$$\beta^4=\rho A\omega^2/EI \qquad (12)$$

The first resonance frequency of the structure is given by [14],

$$\omega_1=\frac{\alpha_1^2}{L^2}\sqrt{\frac{EI}{\rho A_0}} \qquad (13)$$

where $\alpha_1=1.875$, hence $\beta L=\alpha_1(\frac{\omega}{\omega_1})^{1/2}$ (14)

where ω is the vibrating force frequency. The general solution to equation (11) leads to

$$W(x)=A_1\cosh\beta x+A_2\sinh\beta x++A_3\cos\beta x+A_4\sin\beta x \qquad (15)$$

where the coefficients A_1, A_2, A_3, A_4 are determined using four boundary conditions of cantilever beam as follows: the deflection and its first derivative at clamped end is equal to zero. At the free end the moment is equal to zero while the transverse shearing force is equal to the amplitude of the applied force. Solving the system formed by these equations leads to

$$A_2=-A_4=-\frac{F_0}{EI}\frac{1}{(\sinh^2\beta L-\sin^2\beta L)-(\cosh\beta L+\cos\beta L)}$$

$$A_1=-A_3=-A_2\frac{\sinh\beta L+\sin\beta L}{\cosh\beta L+\cos\beta L} \qquad (16), (17)$$

For such a cantilever beam the longitudinal strain is expressed as [Ref.],

$$\varepsilon_x=-z\frac{d^2W}{dx^2} \qquad (18)$$

C. Determination of Bending Moment

This planar beam, i.e., where all forces are exerted in the same plane, is a system of two internal stress components, shear and bending stresses. The bending moment is the result of bending stress of structure is given by [7],

$$M=-\int z\sigma_x dA \qquad (19)$$

According to primary assumptions and simplification of Hook's law, the longitudinal stress is given by [Ref.],

$$\sigma_x=-\frac{E}{1-\nu^2}\varepsilon_x \qquad (20)$$

Substitution in (18) results,

$$\sigma_x=-\frac{E}{1-\nu^2}z\frac{d^2W}{dx^2} \qquad (21)$$

where ν is Poisson' s ratio. For the case of a section made of two different materials, the integral is divided into two parts, one for each elastic material

$$M=-\frac{d^2W}{dx^2}\left[\frac{E_i}{1-\nu_i^2}\left(\int_0^L\int_{z_B}^{z_i}z^2dA\right)+\frac{E_s}{1-\nu_s^2}\left(\int_0^L\int_{z_i}^{z_T}z^2dA\right)\right] \qquad (19)$$

where ν_i and ν_s are Poisson's ratio of ice and substrate, respectively. The terms inside the parentheses represent the moment of inertia of the section about the neutral axis,
The global bending moment is,

$$M=-\frac{d^2W}{dx^2}\left[\frac{E_iI_i}{1-\nu_i^2}+\frac{E_sI_s}{1-\nu_s^2}\right] \qquad (20)$$

The previous relation can be seen from Euler-Bernoulli Beam Theory [14] , which expresses bending moment as a function of deflection. The global bending moment can also be written as,

$$M_G=-EI\frac{d^2W}{dx^2} \qquad (23)$$

where EI is the global flexural rigidity of the structure,

$$EI=\frac{E_iI_i}{1-\nu_i^2}+\frac{E_sI_s}{1-\nu_s^2} \qquad (24)$$

C. Determination of Shear Stress

Assuming the bimorph undergoes small deflections in the linearly elastic region, and has a uniform cross-section. According to Euler-Bernoulli Beam theory shear force can be expressed as the third derivative of transverse deflection,

$$V=-EI\frac{\partial^3 w(x,t)}{\partial x^3}=-EI\frac{d^3W(x)}{dx^3} \qquad (25)$$

The shear stress τ generated from the shear force can now be calculated as [12] [14],

$$\tau = \frac{VQ}{Ib} \tag{26}$$

where Q is the first moment of the area about the neutral axis. For composite areas, i.e., ice and substrate, the first moment of area can be calculated for each part and then added together. The equation for Q in this case is

$$Q = \sum_{j=1}^{n} A_j^* z_{c_j^*} \tag{27}$$

where A* is the area of the part of the cross section that is considered, $z_{c_j^*}$ is the vertical distance from the centroid of the cross section to the centroid of A* as shown in Fig.4 and I is the moment of inertia of the section about the neutral axis.

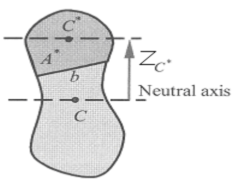

Fig.4. z_{c^*} is the vertical distance from the centroid of the cross section to the centroid of A*.

Substituting (25) in (26) gives the shear stress as a function of bending displacement,

$$\tau = -E\frac{Q}{b}\frac{d^3W}{dx^3} \tag{28}$$

Assuming that before ice detachment, in the linearly elastic region, the displacement and the strain tensor are the same for both substrate and ice at interface, because they are closely linked.

D. Determination of Bending Stress

Bending stress is zero at neutral axis and assumed to increase linearly to a maximum at the outer fiber of the section. Solving the moment equation for stress gives us the bending stress within the ice [7],

$$\sigma_{xi} = -\frac{M_G}{I} z_i \tag{29}$$

where M_G is the global bending moment, z_i is the distance from the neutral axis within the ice, and I is the moment of inertia of the section about the neutral axis of non-homogeneous beam which can be expressed as [7].

$$I = I_i + (E_s / E_i) I_s \qquad (Y_s > Y_i) \tag{30}$$

Substituting (23) in (29) gives the bending stress of ice as a function of bending displacement,

$$\sigma_{xi} = -E\frac{d^2W}{dx^2} z_i \tag{31}$$

The bending stress of the substrate is expressed as,

$$\sigma_{xs} = -\frac{E_s}{E_i}\frac{M_G}{I} z_s \tag{32}$$

z_s is the distance from the neutral axis within the substrate. By substituting (23) in (32) bending stress as a function of deflection is given by,

$$\sigma_{xs} = -\frac{E_s}{E_i} E \frac{d^2W}{dx^2} z_s \tag{33}$$

VI. PVDF ADVANTAGES AND SENSOR EQUATIONS

With the advent of multi-layer materials and the development of active/smart materials, it became possible to use them for studying the mechanical behavior of materials with the help of them. Smart/active materials take input energy in one form then convert it in another form. Like water from a sponge, piezoelectric materials generate charge when squeezed. It can develop an electrical charge proportional to a change in mechanical stress. Piezoelectric film can be used to sense the adhesion force at the ice/substrate interface. Hereafter the advantages of using piezoelectric films (PVDF) as stress sensors will be mentioned and related sensor equations to predict the charge density induced on the piezoelectric film will be explained.

For measurement purposes, using piezoelectric films as sensors has the following advantages. They are thin, light, relatively low in cost, highly sensitive to electrical induction and mechanical load, and they can easily be manufactured into any desired shape. They can also be easily miniaturized, mounted on, and integrated into the structure. Moreover, they are especially interesting due to their high sensitivities to small variations in applied loads although their function depends on temperature variations [11]. Indeed using PVDF films in this configuration allows measuring the combination of shear and bending stresses anywhere at ice/ material interface.

The fundamental piezoelectric coefficients for charge and voltage predict the charge density and voltage developed by the piezoelectric. Piezoelectric charge and voltage coefficients are each assigned two subscripts: one referring to the electrical axis, the other to the mechanical axis. In charge mode the generated charge density is given by

$$\rho = q / A = d_{3n} T_n \qquad n = 1, 2 \text{ or } , 3 \tag{34}$$

where ρ is the charge density developed, q is the charge developed, A is the electrode (piezoelectric film) area, d_{3n} is the Piezoelectric coefficient, n is the axis of applied stress, and T_n is the stress. It can be seen that by measuring the charge developed on the film, the direct measurement of stress, superposition of bending and shear stress will be possible.

$$T = \tau + \sigma \tag{35}$$

Assuming the piezoelectric film dimensions are b as width at y-direction and e as length at x-direction. As it is shown before, the induced stress along the piezoelectric film is a function of x, therefore the distributed electric charge due to induced stress on the piezoelectric film is given by,

$$q = \iint_{area} \rho . dA \tag{36}$$

Substituting (34) and (35) in (36) results in,

$$q = \int_0^b \int_0^e d_{3n}(\tau + \sigma) dx dy \tag{37}$$

The stress distribution along the y-axis is constant, thus

$$q = b.\int_0^e d_{3n}(\tau + \sigma) dx \tag{38}$$

where τ and σ are given by (26), (31), and (33). Therefore the total charge developed on the piezoelectric films can be calculated as first and second derivatives of structure deflection.

VII. CONCLUSION

The analytical solution of deflection of a non-symmetric cantilever bimorph constituted by a layer of ice deposited on an elastic substrate, driven into harmonic vibration is obtained.

The neutral plane position of the structure as a function of ice and substrate thicknesses is obtained. Bending displacement of the structure is derived with the help of Euler-Bernoulli Beam theory. The moment, the shear and bending stress of the structure are obtained as functions of bending displacement. The piezoelectric charge coefficient is used to predict the charge density induced on the piezoelectric film. The discrepancies in Young moduli and the geometric dimensions of the two layers are taken into account in the modeling.

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