Numerical Modeling of Snow Accretion over a Cable Span

Dr P. Fu and Prof. M. Farzaneh
NSERC/Hydro-Quebec/UQAC Industrial Chair on Atmospheric Icing of Power Network Equipment (CIGELE) and Canada Research Chair on Atmospheric Icing Engineering of Power Networks (INGIVRE) at the University of Quebec at Chicoutimi, Chicoutimi, Quebec, Canada,
http://www.cigele.ca

Abstract - **This paper is intended to model the snow accretion process on power transmission cables and wires. In particular, this model is designed to predict the geometry and load of snow accumulations over a cable span. The model takes as input the cable parameters and the meteorological conditions particular to a snow event, namely cable length, diameter and torsional stiffness, snow flake diameter, precipitation rate and air speed. Airflow calculation is performed over the snow-covered cable so as to determine the air pressure and shear stress distribution, which will later be used to evaluate the twisting angle of the cable. The cable surface is partitioned into a number of linear elements so that all the relevant parameters can be evaluated locally. For estimating snow accretion geometry time-dependently, a new algorithm that assumes a locally radial snow growth is implemented. It is believed that a higher degree of accuracy for predicting the evolution of the snow geometry may thus be achieved. The twisting angle along the cable span is determined by solving a set of non-linear torque equilibrium equations. The snow load data calculated from the model is then compared to those obtained by the simulation of certain practical models.**

I. INTRODUCTION

Snow is regarded as an invaluable gift from the Mother Nature. People enjoy snow falls and snow-related activities. Also, snow is a vital source of the pure water on the earth, which is particularly true considering global warming and industrial pollution. However, snow can be a hazard as well, as is the case for snow accumulations on power network structures which may lead to power outages and damage to transmission equipment. Therefore, snow and wind-on-snow loads must be taken into account in the practical design of power network aerial transmission lines in northern regions, thus warranting an in-depth understanding of the phenomenon.

The physics of the process of wet-snow adhesion is not well understood yet (Poots 1996), though the earliest experimental or theoretical studies can date back to dozens of years ago. On the experimental front, researchers (Sakamoto 1993; Admirat 1988) were able to simulate snow accumulations under experimental conditions and then combine the experimentation data with on-site observations to derive certain empirical formulae that can be used to predict parameters like snow density and accretion factor. It should be noted that the snow density and accretion factor are parameters of prime importance, which may vary to a high degree according to the location where it occurs. That explains why the results from Sakamoto and Admirat differ to a high degree. Besides, the number of variables used to determine these parameters in both models are different. For example, Sakamoto's density and accretion factor are function of wind speed and air temperature while Admirat's are function of wind speed only. It should be pointed out that experimental and observational methods are probably the most reliable ways to determine these parameters.

In parallel, a number of researchers (Makkonen 1989; Skelton et al. 1991) have carried out theoretical study in an attempt to create a physical model that can be used to predict snow load and accumulation geometry. Such studies may also be applied to examine the practical models. The physical processes in these models are, however, extremely complex. In general, two types of studies on snow accretion can be found in the literature. The first type consists in practical model studies carried out, for example, by Admirat et al. (1988), Sakamoto et al. (2000) and Makkonen (1989). The snow accretion rate derived from these models is a function of constant parameters such as the accretion factor and density of snow accumulation, thus being possibly considered as static models. Care should be exercised when using these models since they have been calibrated for wet snow episodes in certain regions or countries. The studies concerning the second type of models (Skelton et al. 1991) involve research attempts for dynamic modeling, in which the model parameters were calculated in a time-dependent manner. In these models, the accretion factor is a function of both the meteorological conditions and the shape of the snow accumulation. In the case of transmission lines, however, heat transfer occurring on the snow accumulation, as well as heating due to the electrical current in the cable, should be taken into account.

II. PRINCIPLES

The cable under consideration for the present study was evenly partitioned into a number of sections. The calculation procedures for airflow, snow load and geometry were carried out sequentially for each section. Airflow calculation was performed over the snow-covered cable so as to determine the air pressure and shear stress distribution, later to be used to evaluate the twisting angle of the cable. The local accretion factor was assumed to be governed by the Cosine law, and Sakamoto's density formulae were used to evaluate the snow density. Admittedly, Sakamoto's formulae were calibrated specifically for the snow accretion in Japan and using them elsewhere may cause an overestimation of snow density. The same procedure was repeated throughout all cable sections. The rotation process is assumed to be a static-equilibrium problem, and hence the external torque must be balanced by the internal resistance torque so developed. Accordingly, the procedure for determining the rotation angle was carried out based on the solution of the torque balance equation. Finally, an improved Newton-Raphson method was used to solve the moment balance equation to get the twisting angle for each time step. The snow deposits over the cable surface are assumed to exert negligible resistance to cable rotation.

2.1 Predicting Snow Accretion

The local snow accumulation rate over the cable surface was determined in terms of the precipitation rate and the local surface curvature. In particular, the calculated snow accumulation rate is multiplied by the accretion factor so as to account for the loss of snow accumulation due to bouncing back.

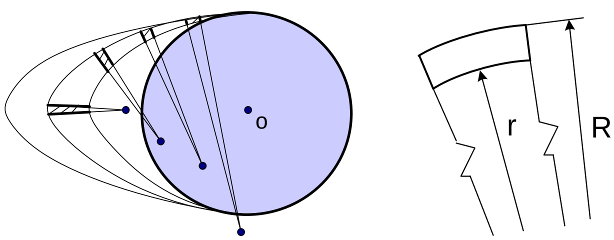


Figure 1 Schematic diagram for evaluating the thickness of a new accumulation layer

For estimating the snow accretion geometry dynamically, a new algorithm assuming local radial snow growth was implemented, as shown in Figure 1. The radius of the local curvature, r, can be determined according to the three adjacent nodes at the location. Then given the snow density, the thickness of the snow layer can be evaluated based on the quantity of snow accumulated during any particular time step, that is R - r. Of course, the density of accumulation plays a vital role for determining the eventual snow shape.

2.2 Predicting the twisting process for the snow-covered cable

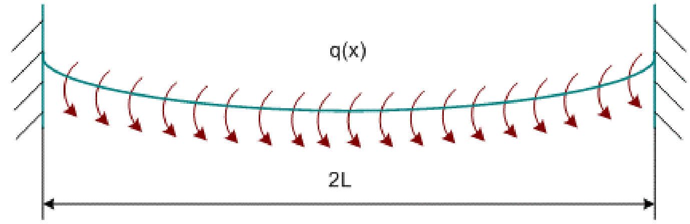


Figure 2 a cable span with snow accumulation

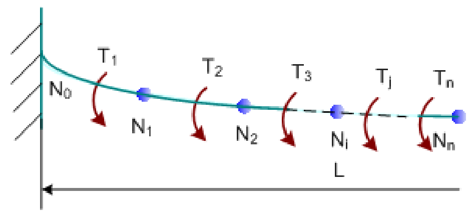


Figure 3 the isolated left half

Figure 2 shows a cable of 2L in length fixed at both ends, which undergoes distributed torsional moments as snow accretes on its surface. For the purpose of calculation, the cable under consideration is discretized into $2n$ elements. Ice mass on any particular element may be considered to be uniformly distributed, provided that its length is sufficiently short. Moreover, the evenly distributed moment on each element may be replaced by a discrete torque, the quantity of which equals the integration of the former distribution. The discrete torque acts on the element with the application line passing through its middle point. A cut is made in the cable mid-span and the left half is isolated, then the free-body diagram can be obtained, as shown in Fig. 3.

At any time step k, the rotation angle of the node, N_i, is equal to the accumulated angles created by the discrete torques $T_1 \ldots T_n$ independently, formulated as follows:

$$\phi_i^k = \frac{0.5T_1^k l}{GJ} + \frac{1.5T_2^k l}{GJ} + \ldots + \frac{(j-0.5)T_j^k l}{GJ} + \ldots + \frac{(i-0.5)T_i^k l}{GJ} + \frac{iT_{i+1}^k l}{GJ} + \ldots + \frac{iT_n^k l}{GJ} \quad (1)$$

or

$$\phi_i^k = \sum_{j=1}^{n} A_{ij} T_j^k \quad (2)$$

where i and j represent the number for nodes and torques, respectively;

$$A_{ij} = \frac{(j-0.5)l}{GJ} \quad \text{for } i>=j$$

or $$A_{ij} = \frac{il}{GJ} \quad \text{for } i<j,$$

where $l=L/n$, representing the length for each element;

J is the polar moment of inertia, G is the shear modulus.

Note that in Eq. 1 T_i is a function of ϕ_i^k, which could be replaced by $\phi_i^{k-1}+\Delta\phi_i$ in the Finite Difference Method (FDM). The term ϕ_i^{k-1} is the rotation angle at the end of the *k-1* step, and thus one known parameter for the subsequent step, *k*. As a result, the equation system for any time step, as specified in Eq. 1, may be evaluated in terms of $\Delta\phi_i$. The external torque for any element, *T*, comprises the torque from air pressure, T_p, air shear, T_f, and gravity, T_g, formulated as follows:

$$T = T_p + T_f + T_g \tag{3}$$

where T_p and T_f can be obtained by calculating the velocity boundary layer (Fu et al. 2007).

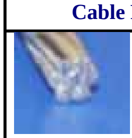
2.3 Solution

The nonlinear system of Equations, as formulated in Eq. 1, may be solved using the multi-dimensional Newton root finding method which, in reality, is an improved Newton-Raphson method in terms of its global convergence attained through more sophisticated implementations. It has been found in the present study that this iterative method is particularly robust. Moreover, the algorithm imposes no specific restriction on the initial guess and its convergence is not excessively sensitive to changes of system parameters. Such a solution yields $\Delta\phi_i$ and also ϕ_i^k from $\phi_i^k=\phi_i^{k-1}+\Delta\phi_i$ for any cable section at each time step. A coordinate translation is then carried out so as to obtain an updated ice shape immediately after the rotation of the cable. The subsequent simulation of snow accretion is performed on the basis of the newly obtained ice shape. In this fashion, the iterations are carried on until the end of the simulation.

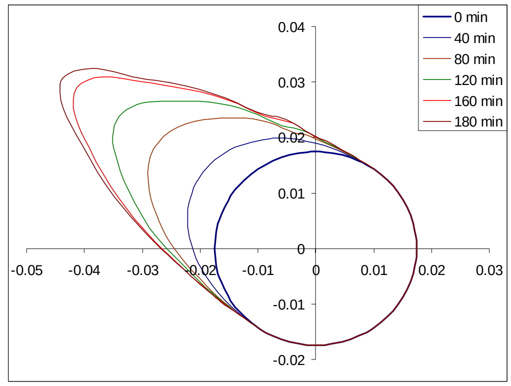
III. SIMULATIONS AND RESULTS

In order to estimate the effects of the cable torsional rigidity on the snow accumulation process, two simulations were carried out separately for non-rotational and rotational cables. In the first batch of simulations, a 35-mm cable is taken and Table 1 shows the relevant conditions. The simulation results for snow accumulation geometry and snow weight over time were obtained and illustrated in Fig. 4

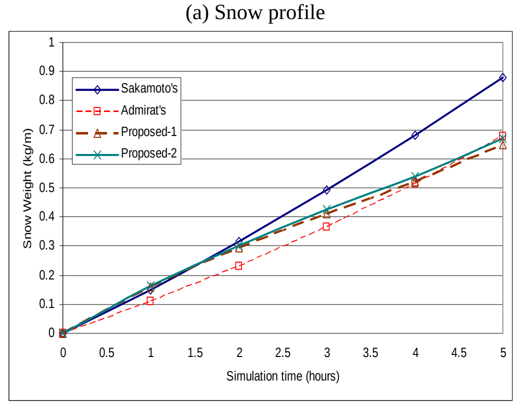
Table 1 Simulation parameters

Cable Parameter			**Model Input Parameters**			
 60 m	**Dia. mm**	**rigidity** N.M^2/Radian	**V0 (m/s)**	**T (°C)**	**Flake Dia. (mm)**	**LWC g/m^3**
	35	Inf.	4	0	10	0.36

The snow profile figure reveals that for a non-rotational cylinder the snow grows outwards making an angle with the incoming wind direction and that the snow shape becomes increasingly aerodynamic. That explains why the present model predicts a gradually decreasing snow growth rate, as can be observed in Fig. 4 (b). But the growth rates in Sakamoto's and Admirat's models tend to be accelerating. This difference may be explained by the fact that the cable under consideration is non-rotational. Despite this difference, the predicted results in the present model display a good consistency with the other two models for most of the simulation time.



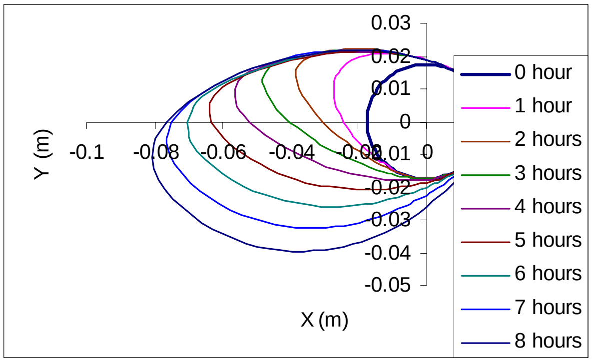
(a) Snow profile



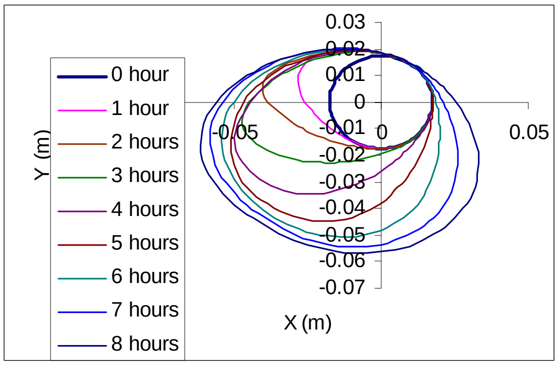
(b) History of snow weight

Fig. 4 simulation results for a non-rotational cable

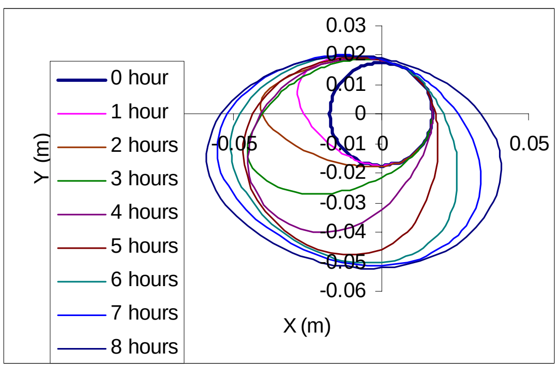
Simulating snow growth on a rotating cable presents a genuine challenge. For the simulation, the half cable span under consideration is partitioned into ten elements, assuming that ice mass is evenly distributed on the cable surface. The rotating cable shares the most parameters with the non-rotating case, except torsional rigidity, the value of which was assumed to be 35 N.M^2/Radian. The output of the simulation is, among others things, data capable of generating ice shapes corresponding to three different locations over the cable span, as shown in Fig. 5. The graphs show sets of accumulation shapes at sections with dimensionless distances of 0.1, 0.5 and 0.9 to the left support. It will be observed that ice tends to grow spirally over all sections during the simulation. After 8 hours of simulation time, the snow accretion at the middle section exhibits a perceivably circular shape while the one close to the left support is still asymmetrically distributed at one side.



x = 0.1



x = 0.5



x = 0.9

Fig. 5 simulation results for a rotating cable

Figure 6 shows the history of the snow distribution on the cable. It can be observed that initially the snow load is distributed evenly on the cable. As the simulation proceeds, more snow load accumulates on the middle sections than elsewhere. However, snow distribution can be considered as uniform on the cable since, according to the same figure, the cable is evenly covered by the maximum snow load over 70 percent of its length.

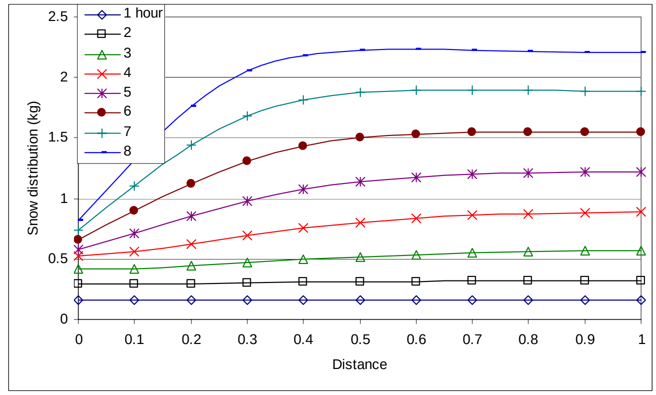


Fig. 6 Snow distribution

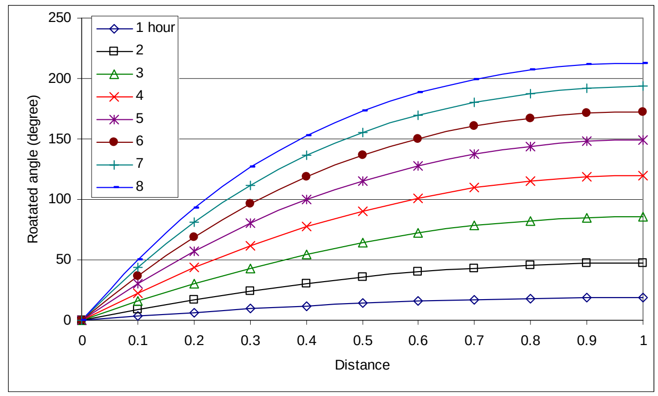


Fig. 7 Angle distribution

Figure 7 shows the time evolution of the cable rotation angle for different cable sections. Not surprisingly, the maximum rotation angle appears in the mid-span. However, the cable twists most at the left support, characterized by the stiff slope and thus by the maximum shear stress. It can be concluded that the cable may be subjected to the greatest torsional fatigue at the same location.

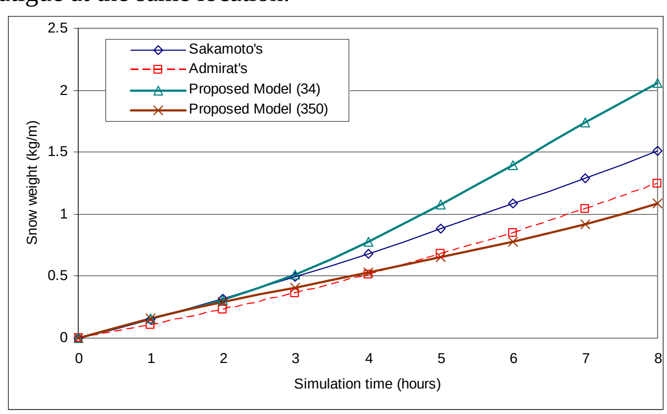


Fig. 8 history of snow weight

The history of the snow-mass accumulation is shown in Fig. 8, in which the mass curve for the soft cable (torsional stiffness: 34 N.M^2/Radian) is found at the top. This means that a soft cable tends to collect more snow than a rigid one of same diameter during the same snow event. It is also observed in Fig. 10 that the curve for the cable (torsional stiffness: 350

N.M^2/Radian) shows a striking similarity with Admirat's model, though diverging gradually from it as the simulation proceeds.

IV. CONCLUSIONS

A model designed specifically for simulating the snow accretion process on transmission line cables was introduced. The new numerical model is able to predict snow geometry, snow-load distribution and twisting angle distribution. Owing to simulations using this model, it is possible to conclude that the rigidity of a cable has a direct influence on the rate of any snow accumulation process occurring on its surface. Given two cables of similar diameter, the soft one tends to collect more snow during a particular ice event. For future studies, a thermal analysis of snow accumulation taking Joule heating into account is recommended.

V. ACKNOWLEDGEMENTS

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