New Cyclic Model of Snow Accretion on Power Cable

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Abstract**—** Cylinder-model have been widely used for the evaluation of the weight as for designing new power cables. This model, however, does not involve the mechanism of dropout of accrued snow. Therefore, it grows continually during meteorological conditions for snow accretion, and reaches maximum weight at the end of the conditions. The weight of accrued snow depends largely upon the so-called snow accretion ratio, which is little based on the physics. The external forces acting the accrued snow, which should play an important role for the shape and weight, are not considered in the model. The development of new model has been expected for the prediction of the weight and for the innovation of the anti-icing devises.

The present study started aiming at the development of physics-based new model. Firstly, the geographical analysis was made for the shape and internal structure of the accrued snow produced in cold laboratory. Secondary, experimental measurement of mechanical strength of wet snow samples was made in cold laboratory. Through those processes, a new cyclic model of snow accretion was constructed repeating three processes, birth, growth, collapse-dropout and then rebirth.

The verification of this cyclic model was made applying to Narude Test Line and two accidents, which occur in two commercial power lines. Time-series calculation of weight of accrued snow was made in use of meteorological data including solar radiation. Good cyclic correspondence was recognized in the comparison between observed and calculated weights of the accrued snow, and the physics-based new cyclic model was clarified to be valid.

I. Introduction

Studies on modeling the snow accretion on power cable have been aimed at the evaluation of accretion weight. In the past models(Kurokawa, 1965 ; Goto, 1976; Wakahama et al., 1977; Skelton and Poots, 1991; Poots,1998; Prodi et al.,1990), it is supposed that once snow accretion starts growing, the weight increases steadily and reaches the maximum until the weather conditions for snow accretion end. There are three weak points in the past models; Firstly the little geometric examination of the section form of the accrued snow which decides the aerodynamic behavior, secondly the poor physical basis of accretion ratio which the weight depends on, and thirdly the luck of physical conditions for breaking the accrued snow on a cable.

In the present study, based on the careful inspection of shape and internal texture of accrued snow samples produced both in nature and in cold laboratory, and also carrying out the physical analysis of accretion coefficient and the experiment on the mechanical strength of wet snow, authors propose new cyclic model for snow accretion exhibited in Fig 1.

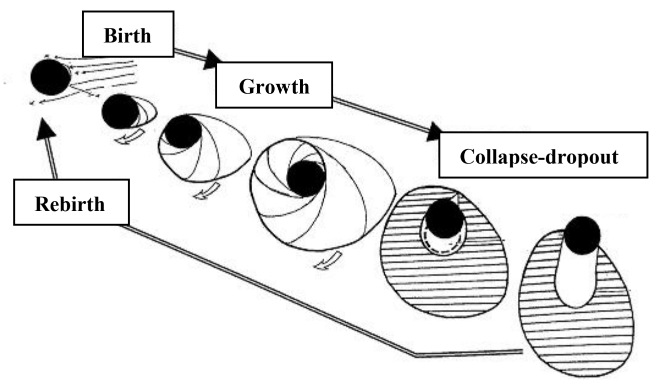

Fig. 1. Cyclic Model of three processes of snow accretion: Birth → Growth → Collapse-Dropout →Rebirth

II. Cyclic Model of Snow Accretion

1. Modeling of Birth Process

Flying snow particles collide, adhere and melt on a cable. Snow accretion starts when the accretion speed exceeds the snow-melting speed. The authors expressed the Birth Model of snow accretion by the following heat budget equation:

$$Q_W < Q_S + Q_R + Q_L + Q_J \qquad (1)$$

As shown in Fig.2,
Q_W: heat for melting all snow particles accrued on cable (J /m/h)
Q_S: sensible heat at cable surface (J/m/h)
Q_R: radiation balance (J/m/h)
Q_L: latent heat (J/m/h)
Q_J: heat generation of cable (J/m/h)

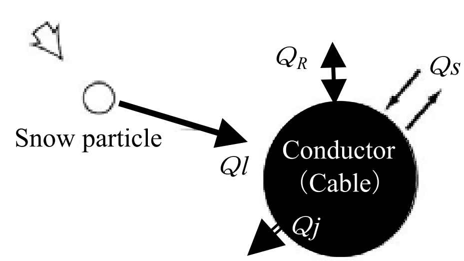

Fig. 2. Heat budget factors of Birth Model

Heat for melting snow particles on a cable

Temperature of snow particles is supposed 0℃. Then, the heat needed for melting all snow particles, which collided to a cable, is expressed by following equation:

$$Q_W = Pc\ 2r(1 - \omega)\ L_W \qquad (2)$$

Pc: snow particles passing through unit area perpendicular to flying direction (kg/m^2/h)
r : cable radius (m)
ω : water content (<1.0)
L_W: latent heat of ice (=3.336×10^5 J/kg)

$$P_c = P\frac{V_f}{V_v} \qquad (3)$$

P : snowfall amount (kg/m^2/h)
V_f: flying speed of snow particle (m/s)
V_v : vertically falling speed of snow particle (= 1.0 m/s)

Where,

$$V_f = \sqrt{V^2 + V_v^{\,2}} \qquad (4)$$

$$V = u \cdot \sin d \qquad (5)$$

V: element of wind velocity vertical to cable (m/s)
u : wind velocity (m/s)
d : angle between wind direction and cable axis (degree)

It may be impossible to make an accurate estimation of water content of flying snow particles. Then, the authors assumed water content ω as function of air temperature as follows:

$$\begin{aligned} \omega &= 0.5T_a\ (0 < T < 2℃) \\ \omega &= 0 \quad (T \leqq 0℃) \\ \omega &= 1 \quad (T > 2℃) \end{aligned} \qquad (6)$$

T_a: air temperature (℃)

Sensible heat at cable surface

Sensible heat transfer from air to cable is expressed by following equation:

$$Q_S = 2\pi r\ \eta\ (T - T_w) \qquad (7)$$

η : heat transfer coefficient (W/m℃)
T_w: cable temperature (℃)

$$h = \frac{\lambda}{2r} \times 0.27 Re^{0.6} Pr^{1/3} \qquad (8)$$

λ : thermal conductivity of the air (=0.0241W/mK)

Re: Reynolds number ($Re = \frac{ur}{\nu}$)

Pr: Prandtl number ($Pr = \frac{C_p \mu}{\lambda}$)

ν : kinematic viscosity coefficient of air (=0.138×10^{-5}m^2/s)
μ : viscosity coefficient of air (=1.76×10^{-5}Pa・s)
C_p: specific heat at constant pressure of air (=1007J/kgK)

Radiation balance

Among short- and long-wave radiations, the long-wave radiation is assumed to be negligibly small under the snow falling weather conditions. The radiation balance is then expressed by following equations:

$$\begin{aligned} Q_R &= Q_R + Q_{ln} \\ &= (1-\kappa)\ Q_r\ 2r \end{aligned} \qquad (9)$$

As shown in Fig. 3、

$$Q_r = Q_t \sin\gamma / \sin h \qquad (10)$$

Q_R : heat absorption of solar radiation (J)
Q_{ln} : heat balance of long-wave radiation (≈ 0)
κ : albedo (≈0.8)
Q_r : element of solar radiation vertical to cable (J)
Q_t : total solar radiation (J)
γ : angle between sunshine and cable axis (degree)
h : solar altitude (degree)

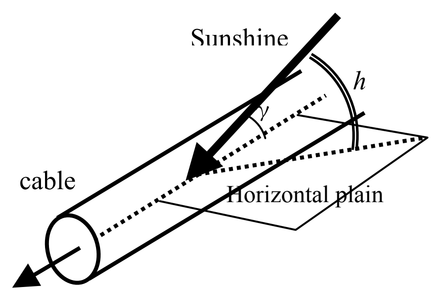

Fig. 3 Angle relation between cable and sunshine

Latent heat

When the snow accretion starts, cable surface is usually wet and the surface water evaporates. The latent heat therefore takes minus or zero value, and is expressed by following equation:

$$Q_L = 2\pi\ r\ D\ L\ (\rho_a - \rho_w) \qquad (11)$$

D: diffusion coefficient (=2.11×10^{-5}m^2/s)
L : latent heat for evaporation (=2.83×10^6 J/kg)
ρ_w: saturation vapor density (kg/m^3)
ρ_a: vapor density of air (kg/m^3)

Heat generation of cable

Joule's heat is caused in the cable, when it is charged with electricity. The heat is expressed by following equation:

$$Q_J = I^2 R \qquad (12)$$

I : electric current (A)
R : electric resistance (Ω /m)

2. Modeling of Growth Process

Exhibited in Fig.4 are the snow accretion samples produced in cold laboratory. The authors made careful inspection of the external shapes and of the internal structures. The shape and structure give the most important and fundamental information giving width and sectional area of accrued snow. Those two key elements for modeling the growth process make the calculation possible on the amount snow particles, which collide and adhere on a cable, and then on the density and weight of accrued snow.

The shape is prescribed by two curves; one is growth line, another is adhesion line. The growth line elongates with the growth of accrued snow. The adhesion line is the surface where snow particles collide and adhere. Those two lines are expressed in Fig.4 with solid and dotted lines, respectively.

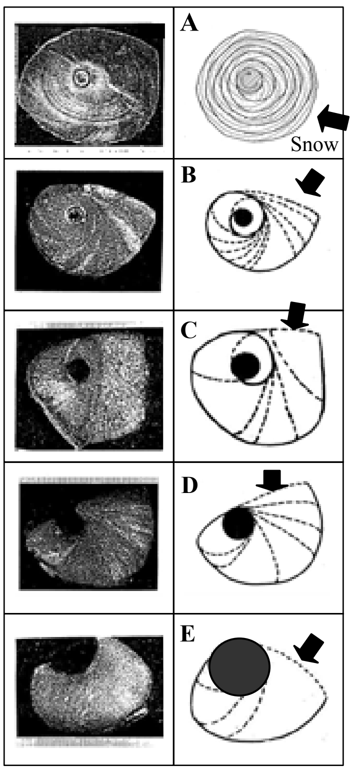

Fig. 4 Pictures and sketches of cross sections of accrued snow samples produced in cold laboratory. Solid line and dotted line exhibit growth line and adhesion line, respectively.
A: cable is not fixed, B～E: cable is fixed (no rotation)

Growth line

The authors approximated the growth line by the following spiral equation.

$$R(\theta) = re^{a\theta(t)} \tag{13}$$

R : moving radius of growth line (m)
r : radius of cable (m)
a : spiral coefficient
θ : angle of moving radius of growth line (degree)

Two examples with different spiral coefficient are shown in Fig.5. Accrued snow becomes cylindrical only when the spiral coefficient is very small. The larger number of spiral coefficient makes more nonsymmetrical spindle shape.

Fig. 5 Two shapes of accrued snow with different spiral coefficient

The spiral coefficient is dependent on the power balance between rotational moment of the accrued snow mass and frictional force at cable surface, The shape is transformed and disturbed due to the plastic deformation, which caused by such external forces as shown in Fig.6.

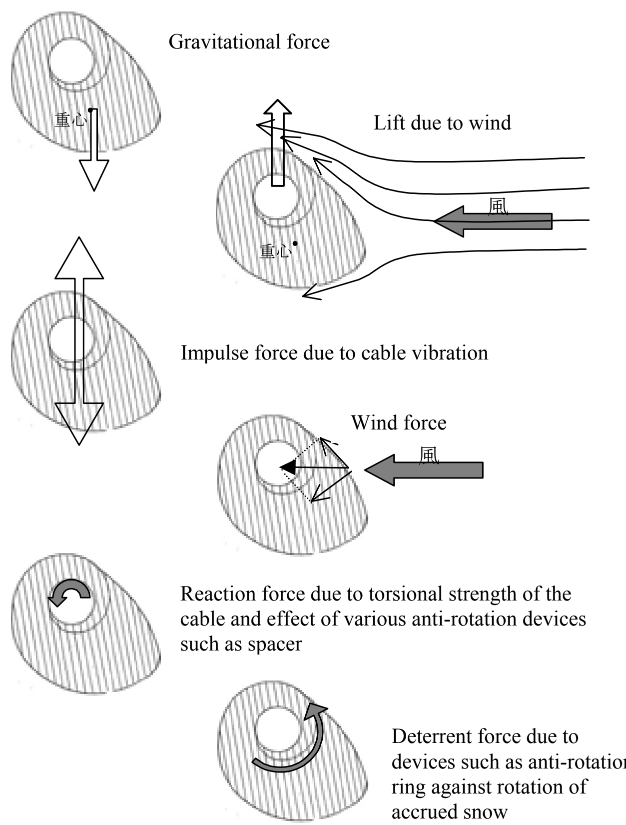

Fig. 6 External forces acting on accrued snow

Adhesion line

The adhesion line can be well approximated by the ellipse, expressed by the following equation.

$$\frac{\cos^2\beta}{R^2(\theta)} + \frac{\sin^2\beta}{R^2(\theta-270)} = \frac{1}{Я^2(\theta+\beta)} \qquad (14)$$

β : angle of moving radius of adhesion line (degree)
Я: moving radius of adhesion line (m)

The center of the ellipse coincides with the center of the cable. The lengths of major axis and minor axis of the ellipse are equal to two moving radius R (**θ**) and R(**θ-270**) of growth line.

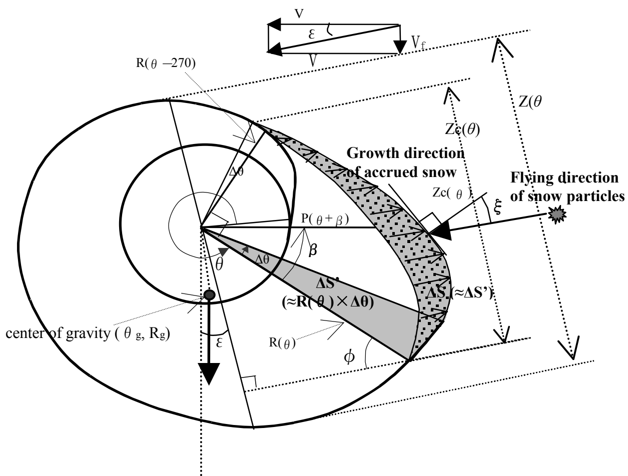

Fig.7 Geometrical relationships between increase of cross sectional area (**ΔS**) and increase of moving radius (Δθ), and between various directions and angles

The angle θ can be calculated under assuming the uniformity of density throughout accrued snow mass, and its dynamic equilibrium around the cable axis.

Snow accretion ratio

Accretion process of flying snow particles is divided into two phenomena of collision and adhesion (Makkonen、2000). As a flying snow particle has inertia, the ratio of collision is close to 1. While the ratio of adhesion is less than 1, and those values play decisive role for the calculation of snow accretion amount.

It is well known that the adhesion of flying snow particles onto many materials as well as cables becomes maximum where the collision angle ξ = 0. Then, the authors distinguished snow accretion coefficient from snow accretion ratio. Accretion ratio is a macroscopic concept for the rate to adhered snow of flying snow particles onto whole cable, whereas accretion coefficient is a microscopic concept for the rate to adhered snow of flying snow particles onto a point of unit area along adhesion line. They are defined by following equations.

$$A = \int^Z \alpha(\xi) \qquad (15)$$

$$\alpha(\xi) = m(\xi)n(\xi) \qquad (16)$$

A: snow accretion ratio (≤ 1)
α: snow accretion coefficient (≤ 1)
ξ : collision angle (degree)
m: collision coefficient (≤ 1)
n: adhesion coefficient (≤ 1)

As a flying snow particle has inertia, the collision coefficient m≈1 throughout the adhesion line. The distribution of accretion coefficient along adhesion line may be derived from the growth direction of accrued snow illustrated by parallel arrows in Fig.7. Then, the snow accretion ratio may be approximated by following equation as a function of adhesion coefficient when ξ = 90.

$$A \approx \frac{n_{90}}{2}\frac{Zc(\theta)}{Z(\theta)} \qquad (17)$$

n_{90}: adhesion coefficient when ξ = 90
Z(θ): collision zone
Zc(θ): adhesion zone

Again, according to Fig.7,

$$Z(\theta) \approx R(\theta+\phi-90)+R(\theta+\phi-270) \qquad (18)$$
$$Zc(\theta) = \{R(\theta)+R(\theta-270)\}\sin\phi \qquad (19)$$

ϕ : angle between moving radius and flying direction of snow particles (degree)

Center of gravity

When growth line of accrued snow is approximated by spiral, and the density is assumed uniform through the snow mass, it is geographically derived that the trajectory of the center of gravity (θ_g , R_g) is also spiral, approximated by following simple equations.

$$\theta_g \approx \theta - 27.6 \qquad (20)$$
$$R_g \approx 0.243\,R(\theta) \qquad (21)$$

When the accrued snow does not freeze on to a cable, and its center of gravity locates just under the cable axis, following equations can be obtained from Fig.7, by which the angle ϕ can be calculated.

$$\theta = \theta_g + \varepsilon + (90 - \phi) \qquad (22)$$
$$\varepsilon = \tan^{-1}(V_f/V) \qquad (23)$$

Weight of snow accretion

The weight of snow particles adhered on a cable per hour is expressed by following equations.

$$\angle W = AM \qquad (24)$$
$$M = P_C\, Z(\theta) \qquad (25)$$

⊿W: weight of snow particles adhered on a cable (kg/m/h)
M : snow particles flying to cable (kg/m/h)

Cross sectional area of accrued snow

Following equation approximates the increase of cross sectional area of accrued snow exhibited in Fig.7.

$$\Delta S \approx \Delta S' \approx R(\theta)\times\Delta\theta \qquad (26)$$

ΔS : increase of cross sectional area in hour (m^2/h)
$\Delta\theta$: increase of moving radius in hour (degree/h)

Water content and density of accrued snow

Water content of snow surface of adhere line is assumed to be equal to that of the flying snow particles, shown in equation (6). Wet density and dry density of the snow surface are calculated by following equations.

$$\rho_w = \Delta W/\Delta S \qquad (27)$$

$$\rho_d = (1-\omega)\,\rho_w \qquad (28)$$

ρ_w: wet density of snow surface of adhere line (kg/m^3)
ρ_d: dry density of snow surface of adhere line (kg/m^3)

3. Modeling of Collapse-Dropout Process

Experiment on breaking strength of wet snow

Shown in Fig.8 is the accrued snow samples, which grew and broke down in cold laboratory. They broke at the thinest part in thickness. In both nature and laboratory, the mechanical strength has not yet been fully measured for the accrued snow.

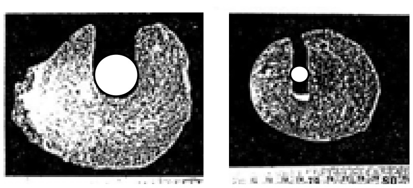

Fig.8 Accrued snow samples, which grew and broke down in cold laboratory

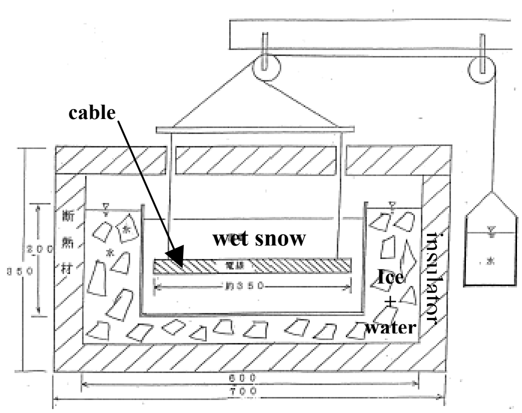

Fig.9 Experimental apparatus to measure the mechanical strength of wet snow

Then, the authors made apparatus shown in Fig.9 to measure the mechanicl strength of wet snw experimantally. Cables with two dufferent radius were buryed in several kinds of wet snow of various different thickness and density.

The cables were slowly pulled up, and the maximum extention force was measured just before the cable started moving upward. Empirically obtained relationshp between maximum force and other elements is shown in Fig.10, and the following equation was obtained.

$$F\max = 1702.4e^{6.32\times10^{-3}\rho_d}\times(H2r) \qquad (29)$$

Fmax: maximum extention force just before cables start moving (kg/m)
H: thickness of wet snow on cable (m)

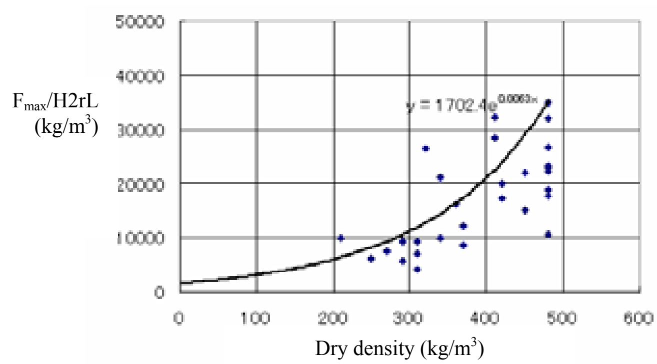

Fig.10 Relationship between Fmax/H2rL and dry density of wet snow (L: cable length)

Collapse-Dropout conditions of accrued snow

Whether accrued snow keeps growing or starts breaking, depends on the balance between breaking strength of accrued snow and external force. If that the maximum extention force is assumed equal to the breaking strength of the wet snow and acting external forces is only weight of accrued snow, the collapse-dropout conditions of accrued snow could be expressed by the following equation.

$$F\max = \sigma_s = W \qquad (30)$$

σ_s: breaking strength of accrued snow (kg/m)
W: weight of accrued snow (kg/m)

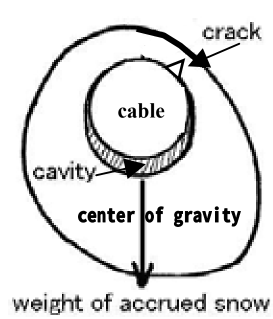

Fig.11 Position of crack where accrued snow starts breaking

Thinest radius of accrued snow is roughly R(θ – 270) as shown in Fig.11, and the thickness is expressed by the following equation.

$$H_{min} = R(\theta - 270) - 2r \tag{32}$$

H_{min} : thickness of thinest radius of accrued snow

With collapse-dropout conditions the cyclic model repeats three processes: birth→growth→collapse-dropout, then rebirth again. Thus, by using the conditions above, the time of collapse-dropout from cable and the maximum weight at that time can be calculated.

3. Verification of the cyclic snow accretion model

The cyclic snow accretion model was applies for the validity to Narude Test Line and two commercially used Tsuruga and Nagahama Lines, where snow accretion accidents occurred recently.

Verification by applying to Narude Test Line

Kansai Electric Power Co. Inc built Narude Test Line on 1979 for the purpose of the snow accretion research. It is a full-scale line with 257m in span of 125 degrees from north, shown in Fig.12, located close to the world heritage Shirakawago along the river Shokawa at 100km south of Toyama City.

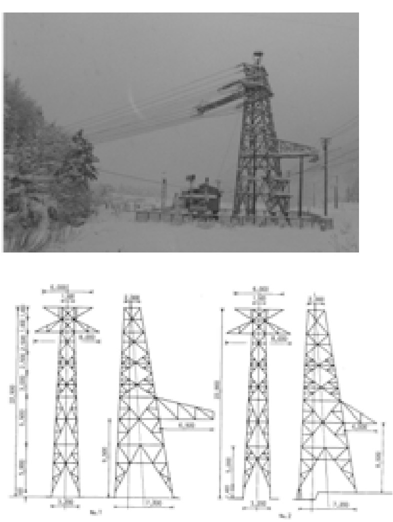

Fig. 12. Narude Test Line

Time series observation data on weight of snow accrued on the various types of cables and on the spot meteorological data such as temperature, wind direction, wind velocity and precipitation were obtained for 10 years since 1981.

Then, the authors tried speculation of solar radiation the noted amount of the accrued snow was recoded in 22-31 Dec. 1984. Shown in Fig.13 are the meteorological data and the measured snow weight accrued on a cable (ACSR 410) in this period. Applying those observation data to the cyclic model, calculation was by Yukino et al. (2007). But they neglected solar radiation, that is Q_t=0 in equation (10), due to unfortunate luck of spot observation data.

Applying the solar radiation data at Toyama Meteorological Station, which is about 50km northwest of Narude Test Line, after the correction by using the correlation between sunshine durations in the Toyama Station and Shirakwa Observatory located in the vicinity of Narude Test Line. Thus obtained solar radiation, which is shown in Fig.13 too, was adopted in equation (10) and the weight of accrued snow was again calculated. The results of two calculations are exhibited in Fig.13 in comparison with the observation data.

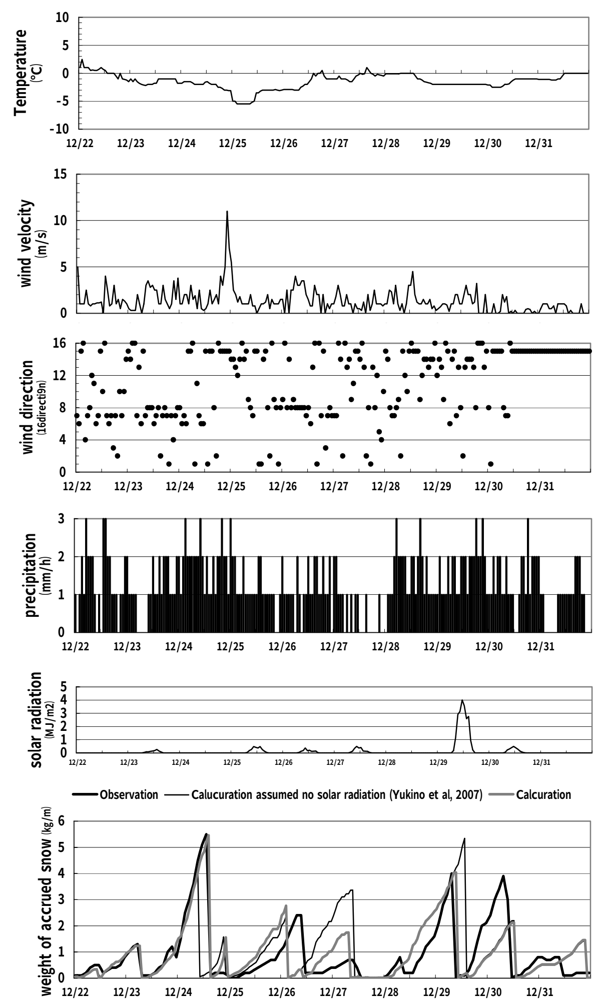

Fig.13Observed spot meteorological data, speculated solar radiation data and comparison of three kinds of accreted snow weight: Observation, calculation assumed no solar radiation (Yukino et al., 2007) and calculation (using speculated solar radiation data)

Good correspondences are found between observation and two calculations. But, better correspondence seems between observation and second calculation in both peak weight and the time when the peak weight occurs. Taking solar radiation into account brought improvement.

In those two calculations, four common assumptions are contained: spiral coefficient of growth line a=0.2, collision coefficient m=1, adhesion coefficient n_{90}=0.8 when ξ = 90 and the center of gravity locating just under the cable axis.

Verification by applying to snow accretion accidents

Snow accretion accidents occurred along two commercially used power line: Tsuruga Line on Dec. 15, 2005 and Nagahama Line on Jan.8, 1996.

Based on the same assumptions above, the time series weight of accrued snow was calculated for two lines. Solar radiation for the Tsuruga Line was speculated from Fukui Meteorological Station and Tsuruga Observatory in the same way as Narude, and for the Nagahama Line solar radiation at Hikone Meteorological Station was adopted.

The results are exhibited in Fig.14 and Fig.15, together with their meteorological data used for the calculation. It is recognized that either of two accidents well coincides the peaks of weight of accrued snow.

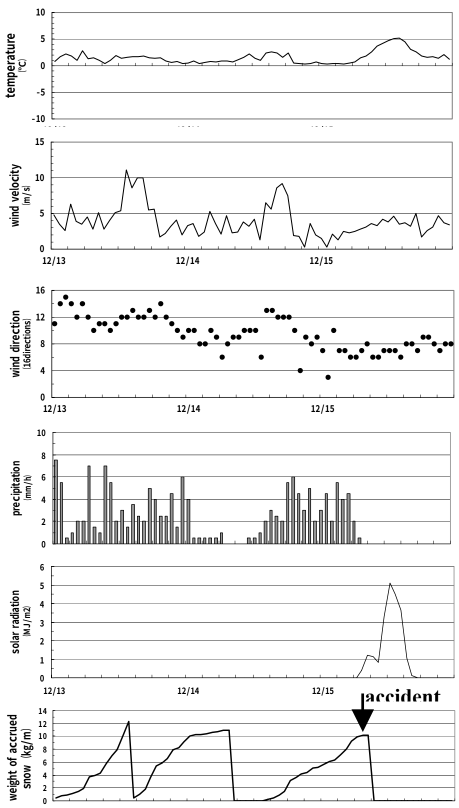

Fig.14 Adoption of cyclic model of snow accretion to the accident of Tsuruga Line at 7:47 on Dec. 15, 2005

III. CONCLUSION

Authors proposed in this paper new cyclic model of snow accretion, which repeats following three processes:

$\boxed{\text{Birth}}\rightarrow\boxed{\text{Growth}}\rightarrow\boxed{\text{Collision-dropout}}\rightarrow$Rebirth

The cyclic model was verified under four assumptions: spiral coefficient of growth line a=0.2, collision coefficient m=1, adhesion coefficient n_{90}=0.8 when ξ = 90 and the center of gravity locating just under the cable axis. Good cyclic correspondence was recognized in the comparison between

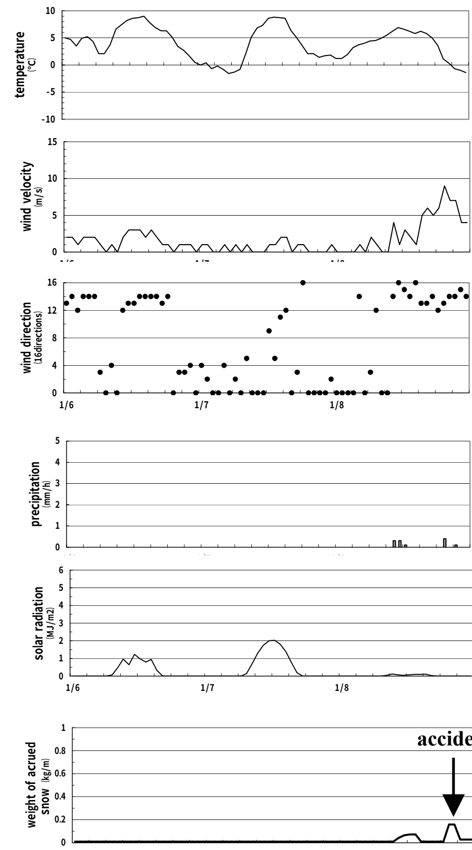

Fig.15 Adoption of cyclic model of snow accretion to the accident of Nagahama Line at 19:45 on Jan. 8,

observed and calculated weights of the accrued snow.

Thus, the physics-based new cyclic model was clarified to be valid. The careful examination of four assumptions will be needed for the further refinement of the model. This model will make it possible to approach the aerodynamic behavior of the snow-accrued power cable too.

Dr. Nishimura, one of the authors of this paper, has passed away in the middle of the present study, which was impossible without his sincere efforts and great contributions. In conclusion, the authors offer this paper to him, and pray his mercy.

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