Mechanical Behaviour of Atmospheric Ice under Different Loading Conditions

M. Kermani¹, M. Farzaneh² and R. Gagnon³

NSERC/Hydro-Quebec/UQAC Industrial Chair on Atmospheric Icing of Power Network Equipment (CIGELE) and Canada Research Chair on Atmospheric Icing Engineering of Power Networks (INGIVRE) http://www.cigele.ca at Université du Québec à Chicoutimi, Chicoutimi, QC, Canada

Institute for Ocean Technology, National Research Council of Canada, St. John's, Canada

Abstract- The mechanical behaviour of atmospheric ice, as an important aspect in the understanding of ice shedding phenomenon, has attracted much attention in recent years. Attempts have been made in this research to shed light on the most significant mechanical properties of atmospheric ice, such as compressive strength, bending strength, effective modulus, and fracture toughness. Atmospheric ice in these tests was accumulated in a closed-loop wind tunnel at -6, -10, and -20° C, with a liquid water content of 2.5 g.m⁻³. Ice samples accumulated at each temperature level were tested at the accumulation temperature, but the ice accumulated at -10° C was also tested at -3 and -20° C. The compressive strength of atmospheric ice increases when test temperature decreases. At low strain rates, this property of atmospheric ice increases as accumulation temperature decreases. The bending strength of atmospheric ice at low strain rates depends on test temperature. The spread in bending strength for different temperatures diminishes as strain rate increases. When compared to the bending strength results for atmospheric ice under static load, the ice shows less resistance against fracture under cyclic load. The bending strength of atmospheric ice under cyclic load decreases with the test temperature. The effective modulus of atmospheric ice usually increases with increasing strain rate. The bending strength of atmospheric ice accumulated at -10° C has been found to be greater than that of ice accumulated at -6 and -20° C. The fracture toughness of atmospheric ice decreases with decreasing accumulation temperature. The value for the ice accumulated at -10° C and tested at the same temperature is 111.17 $KPa\sqrt{m}$. The fracture toughness of atmospheric ice accumulated at -10° C and tested at -20° C increases, owing to higher strength at very cold temperatures.

I. INTRODUCTION

Atmospheric icing of power transmission lines may cause considerable damage to power networks in cold climate regions. Nowadays, many attempts are being made to find efficient methods for removing the atmospheric ice from network equipment, particularly cables and conductors. Consequently, many studies on ice adhesion on substrates, ice shedding and its effects on power network elements have been carried out.

Unlike the other types of ice (lake, river, sea ice, etc.), very few studies on the mechanical properties of atmospheric ice have been reported. Perhaps the most important research on the mechanical properties of atmospheric ice appears in Druez *et al.* (1986) where ice was accumulated at various air temperatures and air speeds, and was tested at the same temperature it had been accumulated. The liquid water content of air (LWC) was set at 0.4 and 0.8 g/m³. The droplet diameter for these two values of LWC was set at 20 and 40 μn , respectively. Also, two strain rates were used for strength tests and several wind velocities for ice accumulation.

Understanding ice behaviour under loading conditions is indispensable for many de-icing studies (e.g. Kalman et al., 2007) and numerical models, including ice shedding models (e.g. Kermani, 2007). This paper describes attempts to shed light on the behaviour of atmospheric ice under different loading conditions and presents a general overview of some mechanical properties (the compressive strength, bending strength, effective modulus and fracture toughness) of atmospheric ice accumulated at three temperatures and tested under various temperatures. A great many studies on the behaviour of other types of ice (particularly freshwater ice) have been published and their results were used for comparison with the results presented herein. Due to space limitations in this paper the descriptions of equipment and apparatus are brief and the very discussions are summarized.

II. SPECIMEN PREPARATION

The ice accumulation conditions for this study were created in the CIGELE² atmospheric icing research wind tunnel, which is a closed-loop (air-recirculated) low-speed icing wind tunnel. Icing conditions, as those encountered during various icing processes in nature, can be simulated in this tunnel. The characteristics of ice accumulation equipment have been discussed in Kermani *et al.* (2007). Air speeds typically leading to natural glaze ice formation ranges from ultra low to medium. In order to make the experimental work more manageable and obtain a more uniform ice layer, air velocity was set at 10 ms⁻¹.

Three ambient temperature values, -6, -10 and -20° C, were selected for the accumulation of atmospheric ice as representative of warm, medium and cold icing conditions.

Liquid water content (LWC) of the CIGELE wind tunnel depends on the difference between air and water line pressures, air speed and the flow rate of water in the supply line. It was calibrated as a function of the aforementioned parameters by Kollár and Farzaneh (2009). LWC for icing

¹ Email : mkermani@uqac.ca Fax number :(418) 5455032

² Email : Masoud_Farzaneh@uqac.ca Fax number :(418) 5455032

³ Email: Robert.Gagnon@nrc-cnrc.gc.ca Fax number : (709) 772-2462

⁴ Industrial Chair on Atmospheric Icing of Power Network Equipment

conditions in nature varies between 0.5 and 10 g.m⁻³, which is within the range of the wind tunnel. During calibration, LWC at the test section of the wind tunnel was measured using the accepted standard technique known as the rotating icing cylinder method (Stallabrass, 1978). Hence, LWC was set at 2.5 g.m⁻³ in order to obtain solid and uniform atmospheric ice.

Atmospheric ice was accumulated on an aluminium cylinder (diameter 78 mm and length 590 mm) placed in the middle of the test section of the wind tunnel and rotated at a constant 2 rpm⁻¹. Before ice accumulation, the cylinder was cleaned with alcohol and set in place for two hours while the system was cooling down. Once accumulation was completed, the cylinder was removed from the test section and the accumulated ice was cut with a warm aluminium blade to avoid any mechanical stress that might cause cracks. The resulting ice slices were then carefully prepared using a microtome. Figure 1 shows the position of the specimens extracted from the accumulated ice on the cylinder, and the load direction in the mechanical tests.



Fig. 1. Specimen position in accumulated atmospheric ice and load direction during test.

Specimen dimensions, as mentioned in ASTM E111-97, were determined by averaging three measurements along the three axes of the samples. In preparing the specimens, the guidelines recommended by the IAHR (International Association of Hydraulic Engineering and Research) working group on test methods (Schwarz *et al.*, 1981) were used.

III. TEST CONDITIONS

The specimens were accumulated at different temperatures and tested at their accumulation temperatures. However, the atmospheric ice accumulated at the typical wintertime temperature (-10° C) was tested at three temperatures -3, -10 and -20° C. The specimens were kept at the testing temperature for two hours before each test.

Table 1 shows the specimen dimensions and test configuration in the different test categories of this study.

A commercial closed-loop, servo-hydraulic material test system (MTS) was used to conduct the tests. During testing, the recording system provides an instantaneous graph which shows the history of both the applied load—or displacement and measured displacement—or load—, so that an exact record of the experiment history is known at the time of testing.

IV. GRAIN STRUCTURE OF ATMOSPHERIC ICE

Thin sections of atmospheric ice accumulated at -6° C (parallel to the cylinder's axis) reveal that the grain size of this

type of ice varies from 0.5 mm to 3 mm, with an average of approximately 1.5 mm. The grain size of atmospheric ice accumulated at -10° C is observed to be considerably smaller than that accumulated at -6° C (approximately 0.5 mm). A significant number of cavities, and possibly cracks, are visible in the ice accumulated at -20° C. The presence of these cavities is attributed to the high freezing rate of the droplets, which prevents them from filling the cavities. Grain size in this type of ice averages less than 0.4 mm, the grain boundaries are more angular than that of the ice accumulated at -10° C, and the cavities are distinctive. The readers are referred to Kermani *et al.* (2007) for more information about the characteristics of ice accumulation equipment and grain structure of these three types of atmospheric ice.

Table 1. Specim	en dimens	sions in	different	tests.
-----------------	-----------	----------	-----------	--------

Test category	Number of tests	Specimen dimensions (mm)		en s (mm)		
		Width	Height	Length	Test configuration	
Compressive strength	181	20	40	45	Uniaxial compression	
Bending strength with static loads	121	20	40	70	Three-point-beam	
Bending strength with cyclic loads	71	20	40	100	Cantilever beam	
Fracture toughness	25	20	40	128	Three-point-beam	

V. COMPRESSION TEST RESULTS

Figure 2 shows the results of uniaxial compression tests of the atmospheric ice that was accumulated at -10° C and tested at -3, -10 and -20° C. In all compressive strength tests, each sample was uniaxially loaded and the axial strain was measured by an extensioneter. In general, the results of these tests are in good agreement with the results of other investigators in many aspects.



Fig. 2. Compressive strength of atmospheric ice accumulated at -10° C and tested at various temperatures. T_r= Test temperature (Kermani *et al.*, 2007).

As reported by many researchers (e.g. Schulson,1990) and made clear in Fig. 2, the compressive strength of atmospheric ice increases with decreasing test temperatures.

As is the case for polycrystalline ice grown in the laboratory, the strength of this ice also increases and then decreases with increasing strain rate.

Figure 3 shows the results of compression tests of the atmospheric ice accumulated at -6, -10 and -20° C and tested at their accumulation temperature. As was mentioned by Wu and

Niu (1994) and demonstrated by Schulson (1990), the compressive strength of ice increases with decreasing grain size. In these tests, as previously mentioned and shown by Laforte and Phan (1983), grain size decreases as accumulation temperature decreases. In Fig. 3, it is observed that the compressive strength of ice increases as the accumulation temperature decreases at strain rates lower than 10^{-3} s⁻¹, because grain size decreases with decreasing temperature. At higher strain rates, however, the compressive strength of ice accumulated at -20° C is less than that for ice accumulated at -10° C. We conclude from this finding that the strength of the ice is more sensitive to the presence of cavities and cracks at higher strain rates than at lower rates, where grain size is the dominating factor.



Fig. 3. Compressive strength of atmospheric ice accumulated at various temperatures and tested at their accumulation temperature. T_a = Accumulation temperature (Kermani *et al.*, 2007).



Fig. 4. Comparison of strength data from the present study with results from other studies (Kermani *et al.*, 2007).

Figure 4 shows the comparison between the present values of compressive strength of atmospheric ice and those of other investigators for fresh water ice. For all fresh water ice tests, the average grain size has been reported to be approximately 1 mm, and test temperature was -10° C. These conditions approximately correspond to the ice accumulated at -10° C and tested at -10° C, as describe herein. The results of the only published report on the compressive strength of atmospheric ice, Druez *et al.* (1986), are included in this figure.

It was shown that air bubbles or voids weaken materials in general. In atmospheric ice, these bubbles can reduce the compressive strength of ice. This is one reason for the differences between the results of this study and those of other researchers for fresh water ice.

In spite of the differences between the data shown by other researchers and the data presented here, the trend of increasing strength with strain rates up to $10^{-3}s^{-1}$ is evident in all the data.

VI. BENDING STRENGTH TEST RESULTS

A. Bending strength tests under static loading

Figure 5 shows the comparative results of bending strength and effective modulus of atmospheric ice accumulated at -6, -10 and -20° C. The results of bending tests on atmospheric ice accumulated at -10° C in Fig. 5 show a clear dependency of bending strength on test temperature for the three lower strain rates.

From the trends observed in Fig. 5, bearing in mind the inherent scatter in the data, the implication is that strain rate and temperature affect the flexural strength of the ice in a small but measurable way. Depending on the temperature, increasing strain rate can either increase or decrease the flexural strength.

As shown in Fig. 5, the bending strength of the ice accumulated at -10° C is higher than that of the two other ice types. The smaller grain size of this ice, as compared to that of ice accumulated at -6° C, as well as the colder test temperature $(-10^{\circ}$ C) explain its higher bending strength. The similar values obtained for bending strength of ice accumulated at -10° C and tested at -3° C and that of ice accumulated at -6° C and tested at -6° C, despite the warmer test temperature of the former, further demonstrates that the bending strength of atmospheric ice accumulated at -10° C is higher than that accumulated at -6° C.



Fig.5. Bending strength of atmospheric ice accumulated and tested at various temperatures (Kermani *et al.*, 2008).

The lack of cavities and pre-existing cracks in the ice accumulated at -10° C, compared to -20° C, results in a stronger structure and higher bending strength for the former, despite the warmer test temperature,. The significant number of cavities, many that have irregular shapes, contribute to the lower strength observed for ice accumulated at -20° C because these cavities are susceptible to stress concentration. It can be seen in Fig. 5 that the average bending strengths of ice accumulated at -10° C and tested at -20° C are greater than the corresponding values for the ice accumulated at -20° C and tested at the same temperature.

Figure 6 shows the comparison between the present values of flexural strength of atmospheric ice (at strain rate of 2×10^{-3}

 s^{-1}) and those of other investigators for freshwater ice and glacier ice. The higher flexural strength of atmospheric ice can be attributed to the lower average grain size in comparison with the ice of other studies.



Fig.6. Comparison of bending strength of atmospheric ice with values of other researchers for fresh water ice and glacier ice (T.I.T= Top In Tension; B.I.T= Bottom In Tension) (Kermani *et al.*, 2008).

B. Bending strength tests under cyclic loading

In order to measure the bending strength of different types of atmospheric ice during a cyclic load with increasing stress amplitude, a load frequency of 0.3 Hz was set for these tests. The direction of bending force was changed twice during each cycle and its amplitude was gradually increased from a value of zero up to that of beam failure with an increasing rate of 2 N/cycle.

Figure 7 shows the comparative results of bending strength of ice accumulated at -10° C and tested at different temperatures under both static and cyclic loads. The difference between the flexural behaviour of atmospheric ice under these two loading modes is distinguishable in that figure. As mentioned above, at higher strain rates under static loads, little or no temperature effect on the bending strength of atmospheric ice is observed. The bending strength of ice under cyclic loads, however, decreases with decreasing test temperatures.

As seen in Fig. 7, at two out of three test temperatures, the bending strength of atmospheric ice under cyclic loads is less than its value under static loads. This reduction has been reported by many other researchers (e.g. Haskell et al., 1996; Haynes et al., 1993; Kerr, 1976). In his research on columnargrain ice, Gold (1972) stated that, at very low temperatures and high strain rates, elastic anisotropy leads to stress concentrations and becomes an important mechanism for crack nucleation. Crack nucleation mechanisms, flaws, microcracks and other defects are more effective in cyclic loads because the reversing bending stress provides identical opportunities for the cracks and cavities at both the top and bottom of the neutral axis of the ice beam to propagate and cause beam fracture. This is not the case for static loading, where the cavities and cracks at the top of the beam are always under compression and in fact don't have any effect on ice fracture.

The difference between the bending strengths of ice accumulated at -10° C and tested at -3° C under cyclic and static loads can be attributed to three possible causes: the inherent scatter in the results of bending strength of ice under both cyclic and static loads; the ductility of atmospheric ice at temperatures close to the melting point, causing finer and

fewer microcracks in each loading cycle than at colder temperatures; and finally, the healing of microcracks at warmer temperatures.



Fig. 7. Bending stress at failure for atmospheric ice accumulated at -10° C and tested at various temperatures under cyclic and static loads in the first scenario (Kermani and Farzaneh, 2009).



Fig. 8. Flexural strength of atmospheric ice accumulated and tested at various temperatures under cyclic and static loads in the first scenario (Kermani and Farzaneh, 2009).

Figure 8 compares the flexural strength of three types of atmospheric ice under cyclic and static loads. The cavities play the same role in weakening the ice accumulated at -20° C under cyclic loads. However, as mentioned previously, they are more effective in cyclic loads because of the reversing bending stress.

VII. EFFECTIVE MODULUS TEST RESULTS

Figure 9 shows the comparative results of effective modulus of atmospheric ice accumulated at -6, -10 and -20° C. As seen in this figure, the effective modulus of all three types of atmospheric ice increases as strain rate increases. This implies that the ratio of "load at failure" to "beam deflection at failure" (F/d) increases with higher strain rates. This is logical because, at a given failure load, beam deflection at low strain rates is higher than at high strain rates. As expected, the effective modulus of ice accumulated at -20° C at higher strain rates is less than that of the two other types, owing to the presence of cavities and cracks in this ice. The effective modulus of ice accumulated at -6° C is higher than that at -20° C because the larger grain size and lack of cavities in the ice accumulated at -6° C make it less deformable (Schulson, 2001).

Furthermore, in spite of the considerable scatter in the data, the effective modulus of the ice accumulated at -6° C is

probably higher than that of the ice accumulated at -10° C. This is so because the values for the ice accumulated at -6° C were greater than or equal to those for the ice accumulated at -10° C throughout the range of strain rates, including the additional data point at 1×10^{-3} s⁻¹ for the ice grown at -10° C. This is due to the larger grain size of the ice accumulated at -6° C, relative to that of the ice accumulated at -10° C.



Fig. 9. Effective modulus of atmospheric ice accumulated and tested at various temperatures (Kermani *et al.*, 2008).



Fig.10. Comparison of effective modulus of atmospheric ice with values of other researchers for fresh water ice (T.I.T= Top In Tension; B.I.T= Bottom In Tension) (Kermani *et al.*, 2008).

Figure 10 shows the comparison between the values of effective modulus of atmospheric ice herein (again accumulated at -10° C and tested at a strain rate of 2×10^{-3} s⁻¹) and those of other freshwater ice research. The differences in the effective modulus issue from dissimilarities between these two types of ice.

VIII. FRACTURE TOUGHNESS TEST RESULTS

Figure 11 shows the results of fracture toughness tests for various types of atmospheric ice. The results show that the fracture toughness of atmospheric ice accumulated at -10° C decreases and then increases with decreasing test temperature.

Figure 11 shows that the average value of fracture toughness of ice accumulated at -10° C and tested at -20° C is 143.28 $KPa\sqrt{m}$, while the corresponding value of the ice accumulated at -20° C and tested at the same temperature is 108.04 $KPa\sqrt{m}$. Comparing the average value of fracture toughness of the ice accumulated at -10° C and tested at -3° C (128.60 $KPa\sqrt{m}$) with the corresponding value of the ice accumulated at -6° C and tested at -6° C (137.75 $KPa\sqrt{m}$), ignoring the small difference in test temperatures and regardless of overlapping error bars, it seems that the fracture

toughness of atmospheric ice decreases with decreasing accumulation temperature. It is obvious that at test temperatures close to the melting point, the ice shows ductile behaviour and is expected that its resistance to crack propagation is expected to increase when the temperature approaches the melting point. The fracture toughness of atmospheric ice accumulated at -20° C and tested at its accumulation temperature is less than other types because the cavities and possibly pre-existing cracks in this ice lower its crack propagation resistance. The same factor observed in the compression tests and the bending tests led to the weakness of this ice.



Fig. 11. Fracture toughness of various types of atmospheric ice. The error bars in this figure correspond to the standard error in the results for the tests conducted with each set of parameters (Kermani & Farzaneh, 2008).



Fig. 12. Comparison of fracture toughness of ice with the results of other research on fresh water ice (Kermani & Farzaneh, 2008).

Fischer and Alley (1995) used a modified ring (MR) test to determine the fracture toughness of granular fresh water ice at -15° C and obtained results of $145 \pm 35 \ KPa\sqrt{m}$. Liu and Miller (1979) obtained the average value of $139 \ KPa\sqrt{m}$ for fresh water ice at -18° C. Goodman and Tabor (1978) obtained the value of $116 \pm 13 \ KPa\sqrt{m}$ for fresh water ice at -13° C. Figure 12 compares the fracture toughness values of atmospheric ice found in this study with those of other research on fresh water ice.

IX. CONCLUSIONS

The compressive strength, bending strength, effective modulus and fracture toughness of atmospheric ice was measured. Using a rotating cylinder in the CIGELE atmospheric icing wind tunnel, ice was accumulated at an LWC of 2.5 gr/m³, wind speed of 10m/s and various accumulation temperatures. The ice accumulated at -10° C was tested at -3, -10 and -20° C. The ice samples accumulated at -6 and -20° C were tested at the same respective temperatures. The results of this research show that the compressive strength of atmospheric ice increases with decreasing test temperatures. Also, it was found that ice strength increases with increasing strain rates up to 10⁻³s⁻¹, and then decreases at higher strain rates. The average bending strength of atmospheric ice under cyclic loads for the ice accumulated at -10° C and tested at -3, -10 and -20° C was found to be 2.81, 2.66 and 2.44 MPa, respectively. Atmospheric ice accumulated at -6 and -20°C and tested at the same temperatures yielded average bending strengths of 2.68 and 1.56 MPa, respectively. At colder temperatures, the flexural strength of the ice under cyclic loads is lower than under static loads. The bending strength was also found to decrease with decreasing test temperatures. The flexural strength of atmospheric ice slightly increases at test temperatures close to the melting point (-3 and -6° C). This can be due to the inherent scatter in the bending test results, to the ductility of the ice at warmer temperatures, and to a possible crack healing mechanism. The flexural strength of atmospheric ice accumulated at -20° C is less than that of the ice accumulated at -6 and -10° C, owing to the presence of cracks and cavities in that ice. The effective modulus of atmospheric ice has been found to increase with increasing strain rates. At higher strain rates, the effective modulus of ice accumulated at -20° C and tested at the same temperature was found to be lower than for the two other ice types. The fracture toughness of atmospheric ice decreases with decreasing accumulation temperature. At very cold temperatures (around -20° C), the fracture toughness of atmospheric ice increases owing to higher strength. However, the fracture toughness of atmospheric ice accumulated at -20° C and tested at its accumulation temperature is less than other types because the cavities and possible cracks in this ice lower its resistance against crack propagation. Owing to structural differences between atmospheric and freshwater ice types, and also to differences in grain size and specimen dimensions in previous studies, some differences between our results and those of other researchers on freshwater ice have been observed.

X. ACKNOWLEDGEMENTS

This work was carried out within the framework of the NSERC/Hydro-Quebec/UQAC Industrial Chair on Atmospheric Icing of Power Network Equipment (CIGELE) and the Canada Research Chair on Engineering of Power Network Atmospheric Icing (INGIVRE) at Université du Québec à Chicoutimi. The authors would like to thank the CIGELE partners (Hydro-Québec, Hydro One, Réseau Transport d'Électricité (RTE) and Électricité de France (EDF), Alcan Cable, K-Line Insulators, Tyco Electronics, Dual-ADE, and FUQAC) whose financial support made this research possible.

XI. REFERENCES

 Dempsey, J. P., Wei, Y., DeFranco, S., Ruben, R., Frachetti, R., "Fracture toughness of S2 columnar freshwater ice: Crack length and specimen size effects- Part1", The Eighth International Conference on Offshore Mechanics and Arctic Engineering, ASME, New York, PP 83-89, 1989.

- [2] Druez. J. Nguyeh, D.D. and Lavoie, Y., "Mechanical properties of atmospheric ice", Cold Regions Science and Technology, Vol. 13: 67-74, 1986.
- [3] Fischer, M.P. and Alley, R.B., "Fracture toughness of ice and firm determined from the modified ring test", Journal of Glaciology, Vol. 41, No 138, PP 383- 393, 1995.
- [4] Gagnon, R.E. and Gammon, P H, "Characterization and flexural strength of iceberg and glacier ice", Journal of Glaciology, Vol. 41, No. 137, PP 103-111, 1995.
- [5] Gold, L. W. "The process of failure of columnar-grained ice", Philos. Mag. A. 26(2), PP: 311-328, 1972.
- [6] Gow, J. A. and Ueda, T. H., "Structure and temperature dependence of the flexural properties of laboratory freshwater ice sheets", Cold Regions Science and Technology, Vol. 16, PP 249-269, 1988.
- [7] Haskell, T.G., Robinson, W.H., Langhorne, P.J., "Preliminary results from fatigue tests on in situ sea ice beams", Cold Regions Science and Technology, Vol. 24, PP:167-176, 1996.
- [8] Haynes, F. Donald, Kerr, Arnold D. and C.R. Martinson, "Effect of fatigue on the bearing capacity of floating ice sheets", Cold Regions Science and Technology, 21, PP: 257-263, 1993.
- [9] Kalman, T., Farzaneh, M., McClure, G., "Numerical analysis of the dynamic effects of shock-load-induced ice shedding on overhead ground wires", Computers and Structures, Vol. 85, no 7-8, pp.375-384, 2007.
- [10] Kermani, M, "Ice shedding from cables and conductors a model of cracking activity of atmospheric ice in brittle regime", Ph.D. thesis, University of Quebec at Chicoutimi, 2007.
- [11] Kermani, M., Farzaneh, M., "Fracture toughness of atmospheric ice", Proceedings of 19th IAHR International Symposium on Ice, Vancouver, British Columbia, Canada, 2008.
- [12] Kermani, M. and Farzaneh, F., "Flexural and low-cycle fatigue behaviour of atmospheric ice", Journal of Materials Science, Vol. 44, pp: 2497-2506, 2009.
- [13] Kermani, M., Farzaneh, M., Gagnon, R. E., "Compressive strength of atmospheric ice", Cold Regions Science and Technology, Vol. 49, 195–205, 2007.
- [14] Kermani, M., Farzaneh, M., Gagnon, R. E., "Bending strength and effective modulus of atmospheric ice", Cold Regions Science and Technology, Volume 53, Issue 2, PP. 162-169, 2008.
- [15] Kerr, Arnold D. "The bearing capacity of floating ice plates subjected to static or quasi-static loads", Journal of Glaciology, Vol. 17, No. 76, PP: 229-268, 1976.
- [16] Kollár, L. E. and Farzaneh, M., "Spray Characteristics of Artificial Aerosol Clouds in a Low-Speed Icing Wind Tunnel", Atomization and Sprays, Vol. 19, No. 4, PP: 389-407, 2009.
- [17] Laforte, J. L. and Phan L., "Microstructure of ice accretions grown on aluminum conductors", Journal of Climate and Applied Meteorology, 22 (7), 1175-1189, 1983.
- [18] Lavrov, V.V., "Deformation and strength of ice", Transl. from Russian, Natl. Sci. Found., Israel Program for Scientific Translation, Jerusalem, 1-164, 1971.
- [19] Liu H.W and Miller K.J, "Fracture toughness of fresh water ice", Journal of glaciology, Vol. 22, PP 135-143, 1979.
- [20] Schulson, E. M, "The brittle compressive fracture of ice", Acta metal Mater, 38 (10): 1963-76, 1990.
- [21] Schulson, E. M., "Brittle failure of ice", Engineering fracture mechanics, 68, PP 1839-1887, 2001.
- [22] Schwarz, J., Frederking, Gavrillov, R., Petrov, I., Hirayama, K., Mellor, M., Tryde, P. and Vaudrey, K., "Standardized testing methods for measuring mechanical properties of ice", Cold Regions Science and Technology, Vol 4: 245-253, 1981.
- [23] Stallabrass, J. R., "An appraisal of the single rotating cylinder method of liquid water content measurement". National Research Council of Canada, Division of Mechanical Engineering, Rep. LTR-LT-92, 1978.
- [24] Timco, G. W. and Frederking, R. M. W., "The effects of anisotropy and microcracks on the fracture toughness K_{IC} of freshwater ice", Proceedings of the 5th International OMAE Symposium, Tokyo, Japan, Vol. IV, PP 341-348, 1986.
- [25] Wu, M. S., Niu, J., "Micromechanical prediction of the compressive failure of ice: Model development", Mechanics of materials, Vol. 20: 9-32, 1994.