Shape Characteristic of iced conductor

Fan Songhai\textsuperscript{1, a}, Liu Ping\textsuperscript{1,b}; Li Jing\textsuperscript{1,c}, Jiang Xingliang\textsuperscript{2,d}

\textsuperscript{1} Sichuan Electric Power Science Research Institute, Chengdu 610072, Sichuan Province, China
\textsuperscript{2} State Key Laboratory of Power Transmission Equipment & System Security and New Technology, Chongqing University, Chongqing 400044, China
\textsuperscript{a}fansonghai@126.com, \textsuperscript{b}liup72@sina.com, \textsuperscript{c}lijing4267@163.com, \textsuperscript{d}xljiang@cqu.edu.cn

Abstract: The cross-section shape of iced conductor played an important role in estimating ice-melting time or ice-melting current. So it is of great significance to investigate the shape characteristic of iced conductor. On the basis of a lot of investigations on the field experiments, this paper put forward an ellipse-shaped model to describe the shape characteristic, and classified the iced conductors into three species, including concentric circle, eccentric circle and eccentric ellipse. The analytic results show that the cross-section shape of iced conductor meets a hypothesis, namely that the maximum thickness of ice is on the upwind side of conductor, while the minimum on the downwind side.

Key words: conductor; ice; ice storm; stiffness

1 Introduction

In early 2011, a large scale ice storm took place in the southern part of China. It has continued more than 20 days as at the end of January. Many transmission lines accumulated ice with thickness over 10mm, and many of them were forced to be out of service. Three men were died for preventing transmission lines from damaging by the ice storm. Similar ice storm took place in the southern part of China in early 2008, which had made at least 7541 transmission lines above 10kV and 859 substations above 35kV out of service\textsuperscript{[1]}, as a result, it brought about a direct economic loss more than 130 billion RMB, and impacted people's lives and works seriously.

After the ice storm of 2008, both the State Grid Corporation and the Southern Power Grid of China have focused on the ice-melting technology, and developed a lot of DC ice-melting equipments, which played an important role in preventing power lines from damaging by ice storm\textsuperscript{[2]}. It is necessary to estimate ice-melting time or ice-melting current before ice-melting is put in practice on transmission lines. Almost all of the actual methods to estimate ice-melting time or ice-melting current have a common hypothesis, namely thinking that the iced conductor has a circular cross-section\textsuperscript{[3-5]}. Unfortunately, this hypothesis is often inconsistent with most actual situations. Literature \textsuperscript{[6]} classified the cross-section shape of iced conductor into seven species, such as circle-shaped, ellipse-shaped, fan-shaped, comb-shaped, needle-shaped, box-shaped and wave-shaped. For glaze iced conductor, the circle-shaped and the ellipse-shaped cross-section of iced conductor is more familiar. An additional error will produce when the estimating method of ice-melting time for circle-shaped iced conductor is applied to that for non circle-shaped iced conductor, and it may be very great under some certain conditions. So it is of great significance to investigate the shape characteristic of iced conductor. After a lot of investigations on the Xuefeng Mountain Natural Icing Station, this paper put forward an ellipse-shaped model to describe the shape characteristic.

2 Shape characteristic of iced conductor

When a droplet moves with air-stream towards the conductor, its trajectory is determined by the forces of aerodynamic drag and inertia \textsuperscript{[7]}. If inertial forces are small, then drag will dominate and the droplets will follow the streamlines of air closely (Fig. 1). Since air must go around the object, the droplets will, in this case, also tend to do so \textsuperscript{[8]}. For large droplets, on the other hand, inertia will dominate and the droplets will tend to hit the object, as results causing ice to grow on the upwind side of conductor.
The cross-section shape of iced conductor depends on many factors such as wind velocity, wind direction and droplet size as well as conductor structure. When these are known, the cross-section profile of iced conductor is mainly determined by conductor stiffness [9]. As shown in Fig.2, gravity torque \(M_1\) is produced by the ice depositing on the upwind side of conductor, and it makes conductor twist, this results in a contrary torque \(M_2\) due to the stiffness of conductor [10]. The iced conductor will turn when \(M_1 > M_2\), and not turn when \(M_1 \leq M_2\). For small stiffness conductor, the cross-section profile of iced conductor is approximatively circular as a result of the turning of conductor, but for large stiffness conductor, the ice grows mainly on the upwind side of conductor. More common situations are that the cross-section of icing conductor likes an ellipse with the conjunct effect of \(M_1\) and \(M_2\) (Fig.2).

According to the results of observing ice on power lines [6], we classified the iced conductor into circle shape, ellipse shape, sector shape, wavy shape, comb shape and so on by its cross-section profile. In Xuefeng Mountain Natural Icing Station, which was built by Chongqing University in 2009, we have found that the cross-section profiles of wet-iced conductors (some of them presented in Fig.3) were approximately elliptical in most instances, and they have a common characteristic that the maximum thickness of ice was on the upwind side of conductor, while the minimum on the downwind side.

### 3 Mathematical mode of iced conductor

In order to describe the characteristic of cross-section profile of iced conductor, two dimensionless numbers are defined as

\[
\begin{align*}
\delta &= \frac{(b - a)}{b} \\
\zeta &= \frac{(d_{\text{max}} - d_{\text{min}})}{d_{\text{max}}}
\end{align*}
\]  

(1)
where $\delta$ denotes ellipticity of iced conductor; $\zeta$ denotes eccentricity of iced conductor; $a_i$ denotes the short axis length of elliptic iced conductor, m; $b_i$ denotes the long axis length of elliptic iced conductor, m; $d_{\text{max}}$ denotes the maximum ice thickness on the upwind side of conductor, m; $d_{\text{min}}$ denotes the minimum ice thickness on the downwind side of conductor, m.

$$x^2 + y^2 = R_c$$  

The second derivative of equation (5)-(6) can be expressed respectively as

$$\frac{d^2 x}{dy^2} = \frac{y^2 b_i^4}{a_i(a_i^2 + y^2)^{1/2}} - \frac{b_i^4}{a_i(a_i^2 + y^2)^{1/2}}$$  

$$\frac{d^2 x}{dy^2} = -\frac{y^2}{(R_c^2 - y^2)^{3/2}} - \frac{1}{(R_c^2 - y^2)^{1/2}}$$

According to the hypotheses (II), we know that the curvature of interface $\Gamma_{01}$ is always greater than that of interface $\Gamma_{13}$, which means that the value of equation (7) is always greater than that of equation (8) at any value of $y$. The value of equation (7) meets its minimum when $y=0$, so

$$\frac{b_i}{a_i} \leq \frac{1}{R_c} \quad (9)$$

By equation (1) and (3) substituting into equation (9), equation (9) can be expressed as

$$\delta \leq 1 - \left( \frac{R_c}{R_{eq}} \right)^{2/3} \quad (10)$$

(2) The ice thickness is the thinnest on the downwind side of conductor

The polar forms of equation (5)-(6) can be expressed respectively as

$$r^2 \cos \theta + b_i d_{\text{min}} = \frac{r^2 \sin \theta}{a_i} + 1 = 1$$

$$r = R_c$$

where $r_c$ denotes polar radius of interface $\Gamma_{01}$, m; $r_c$ denotes polar radius of interface $\Gamma_{13}$, m.

According to equation (11), ice thickness can be expressed as

$$\frac{2b_i}{d_i(\theta)} = r_c - r_c$$

where $d_i(\theta)$ denotes ice thickness, m.

From Fig.3 we know that the $d_i(\theta)$ gets its minimum ($d_{\text{min}}$) when $\theta=0$, namely

$$d_{\text{min}} = \frac{a_i^2 - b_i R_c}{b_i} \quad (13)$$

By equation (1) and (3) substituting into equation (13), equation (13) can be expressed as

$$\zeta \geq \frac{2R_c \delta(-2 + \delta)}{R_c \sqrt{1 - \delta} + R_{eq}(\delta^2 - 2\delta - 1)} \quad (14)$$
If the ice sleeve is not concentric with the conductor, we say that the iced conductor is eccentric; on the contrary, we say that it is concentric. The circle and ellipse are respectively used to describe the cross-section profile of iced conductor. According to the values of \( \delta \) and \( \zeta \), the cross-section profile of iced conductor can be classified into three class types (Table.1), namely concentric circle, eccentric circle and eccentric ellipse. In Table 1, \( \alpha \) and \( \beta \) denote respectively as following

\[
\begin{align*}
\alpha &= 1 - \left( \frac{R}{R_{eq}} \right)^{2/3} \\
\beta &= \frac{2R_{eq}\delta(-2 + \delta)}{R_{eq}\sqrt{1 - \delta + R_{eq}(\delta^2 - 2\delta - 1)}}
\end{align*}
\]

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>( \zeta )</th>
<th>cross-section profile</th>
<th>sketch</th>
</tr>
</thead>
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<td>0</td>
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<td><img src="image" alt="Concentric Circle" /></td>
</tr>
<tr>
<td>0</td>
<td>0 &lt; ( \zeta ) \leq 1</td>
<td>eccentric circle</td>
<td><img src="image" alt="Eccentric Circle" /></td>
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<tr>
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<td>( \beta \leq \zeta \leq 1 )</td>
<td>eccentric ellipse</td>
<td><img src="image" alt="Eccentric Ellipse" /></td>
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</tbody>
</table>

4 Conclusions

(1) The results on the field experiments show that the cross-section shape characteristic of iced conductor can be described by ellipse model;

(2) According to the shape characteristic of iced conductor, the cross-section shape of iced conductor can be classified into concentric circle, eccentric circle and eccentric ellipse.

References


